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“DIGITAL SIGNAL PROCESSING – PRACTICAL TECHNIQUES, TIPS, AND TRICKS”

ADDING (INJECTING) NOISE TO IMPROVE RESULTS.
DITHERING
DITHERING -1

- Dithering comes from the word “Didder” meaning to tremble, jitter, or shake. Dithering is used in various applications to reduce distortion and quantization errors of low-amplitude signals. You may have already used Dithering without realizing it!

- When a mechanical watch stops ticking it can sometimes be started again by tapping it to “un-stick” the mechanism. This can also be done with older analog meters to get a more accurate reading. Whenever we tap a mechanical measuring instrument we are hoping to get it “unstuck” and provide a more accurate reading by adding some noise (the tapping).
DITHERING -2

• Complicated mechanical bomb-sights proved to be more accurate in combat than in the lab. This surprising result was verified when small vibrators were attached to these mechanical computers thus unsticking moving parts and allowing them to work as well on the ground as in the flying bombers.

• Adding noise to a system doesn’t seem like a good idea at first but it is extremely useful in randomizing quantization errors and allowing extremely small signals to be detected. It can smooth banding in images and is often used in mastering digital audio. Dithering is also used in Analog to Digital Converters (ADC).

• Other applications include finance and medicine.
DITHERING DEMONSTRATION
Here is a conceptual demonstration of Dithering. Assume we have a system that quantizes signal values into integers—in other words it rounds up or down to the nearest integer. Assume for this simple case all our input values are at a constant 3.4 as shown.

Although this case is simplified for instructional purposes, long periods where the data value stays the same or approximately the same are not uncommon.
ORIGINAL SIGNAL AT CONSTANT 3.4
• Signal now “quantized” to nearest integer, “3”.
• Significant quantization error of 0.4 (3.4 - 3).
• Thus the error is $0.4/3.4 = 0.1176$ or almost 12%
PSEUDO-RANDOM NOISE CENTERED AT ZERO
ORIGNAL SIGNAL (ALL 3.4) WITH NOISE "ADDED" (some noise will reduce values)
• Noisy signal “quantized” to nearest integer, 3 OR 4
• Average value (mean) = 3.4219. Much closer to 3.4 than “3”. Quantization error = 3.4219 – 3.4 = 0.0219
• Error = 0.0219/3.4 = 0.0064 or < 1%. Noise helped!
• In digital video compression an area of sky that has little variation in intensity (and/or color) can be represented by a single value rather than separate values for each pixel.

• An adjacent area with a slightly different intensity can also be represented by a slightly different value thus allowing further compression.

• The problem arises that when the picture is reconstructed the human eye can see a border or “band” where the values changed slightly as shown in the next slide.

• By adding a small amount of noise the borders or “bands” are no longer noticeable.
ANNOTATION SHOWS WHERE TO LOOK IN THE NEXT SLIDE
With no abrupt change between the two values, human eye does not notice edges.
RESONANCE
DEMONSTRATION OF METHOD TO CAREFULLY ADD (INJECT) NOISE TO A VERY SMALL SIGNAL THAT IS BELOW THE DETECTION THRESHOLD TO DETERMINE THE FREQUENCY.
• Tsamp = 1 sec, Freq = 1/32 Hz = 0.03125 Hz, Nyquist = 0.5 Hz
• For instruction we use Sec/Hz. Can use Msec/KHz, uSec/MHz, etc.
AGAIN WE HAVE 1 CYCLE/32 SECONDS OR 0.3125 HZ
CLOSE-UP OF ORIGINAL SIGNAL IN HZ

AGAIN WE SEE 0.3125 HZ
MINISCULE SIGNAL WOULD BE IN BIN 64

- THIS SIGNAL IS PRESENTLY UNDETECTABLE
- IF WE COULD DETECT IT, THE PEAK WOULD BE IN BIN 64
- **ADD PSEUDO-RANDOM NOISE FOR SIMULATION**
- **NOISE IS RANDOM IN TIME AND FREQUENCY BUT TINY SIGNAL IS CONSTANT IN FREQUENCY**
- **MAY HAVE MILLIONS OF SAMPLES IN REAL WORLD (PROCESSING GAIN) BUT CAN SHOW METHOD HERE WITH FEWER SAMPLES AND LOWER THRESHOLD**
“White noise” in the sense that it has all the frequencies (like white has all the colors).
NOISELESS SIGNAL (Nothing above Det. Thresh.)

ORIGINAL TINY SIGNAL IS WELL BELOW DETECTION THRESHOLD
NO POINTS DETECTED WITH ORIG SIGNAL
SIG WITH 1X NOISE (Nothing above Det. Thresh.)
SIG WITH 2x NOISE (SOME POINTS DETECTED)
FROM 2048 POINTS (SIGNAL + NOISE) THERE ARE 30 POINTS WITH A MAGNITUDE OF 5.0 OR MORE.
In this close-up we can see clearly the peak at bin 64.

Software has also found the location and magnitude of 2 largest peaks.

Problem is that the “signal peak” at bin 64 is only slightly greater than a noise peak at bin 773. The highest peak is only 1.0523 times the next larger peak. Let us try again with more noise.
ORIGINAL SIGNAL WITH 3x INJECTED NOISE
DETECTABLE POINTS WITH 3 TIMES NOISE
• As with 2x noise case top peak at 64 but the 2\textsuperscript{nd} highest peak has changed location from 773 to 634.
• We can see that the ratio of highest peak (original signal) increased from 1.0523 to almost 1.2.
• It seems adding more noise gave better results.
• See if this is the case as we add even more noise.
Ratio is higher at 1.3. Tiny signal freq easily discernable.
CLOSE-UP OF FFT OF SIGNAL WITH 5x NOISE

Ratio reduced from about 1.3 to about 1.2
CLOSE-UP OF FFT OF SIGNAL WITH 6x NOISE

Ratio getting lower
Ratio getting still lower
Miniscule signal is now 2\textsuperscript{nd}-largest peak with 8x noise
SIGNAL WITH 9x NOISE

SIGNAL WITH 9 TIMES NOISE

THRESHOLD SET AT 5

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CLOSE-UP OF FFT OF SIGNAL WITH 9x NOISE

SOI SWAMPED
• We were first able to detect the frequency of our signal when 2x noise was added to our original signal.
• The best result was when 4x noise was added.
• The most noise we could add to the original signal without beginning to swamp the system was 7 times noise.
• In practice then, a method to find the optimum noise then is to inject a gradually increasing amount of noise into the system. We notice when a peak consistently appears when noise is added and when it goes away. We then back off on the noise injection to a midpoint.
• In this case the midpoint was between 3x and 5x noise.
• Remember that in the real world the noise will be constantly changing. This means that the frequency peaks will also be constantly changing. What we look for is the peak that stays at a constant frequency in the FFT.

• The author has used this method with molecular level miniscule signals that stay at a constant frequency for long periods of time but are below the detection threshold.

• As a small amount of noise is “injected” peaks begin to appear at random frequencies. As more noise is injected we begin to detect a constant frequency peak from the original signal.
SIMULATION OF SCOPE SCREEN WITH 4x NOISE
STOCHASTIC RESONANCE
• Stochastic Resonance is somewhat similar to dithering or simple noise injection. It is used to detect very weak signals. Like dithering, random (stochastic) “white”* noise is added to the signal.

• Listen to some of the strings of the 88 notes on an acoustic piano vibrate after the author speaks the words “Heh” “Low” (Hello) into it.

• *White Noise is all the frequencies. Some background noise machines used in therapist’s offices to mask conversations emit “pink” noise. Just as the color pink has more lower-frequencies than white noise, pink noise has more of the soothing lower-frequency bass tones.
THE WORD “HELLO” IS SPOKEN INTO AN ACOUSTIC PIANO CAUSING SOME OF THE STRINGS TO VIBRATE.

WE NEXT LISTEN TO JUST THE STRINGS FOR “HELLO”

THE WORDS “HEH” AND “LOW” ARE SPOKEN AND WE LISTEN TO JUST THE STRINGS THAT ARE EXCITED. WE CAN ALMOST DISCERN THE WORD “HELLO”.

THIS IS A SIMPLE “AUDIO FOURIER TRANSFORM” – OR “AUDIO SHORT TIME FOURIER TRANSFORM” (STFT)
• Bi-Stable systems are common. Ordinary light switch is either on or off. Other binary systems.
• Other examples include atomic/molecular energy levels, electronic Flip-Flops, and Multivibrators/
• Consider a potential energy graph as shown on next slide. Potential energy is given by

\[ y = \frac{x^4}{4} - \frac{x^2}{2} \]
POTENTIAL ENERGY EXAMP (Bi-Stable)

• Slope is thus given by
  \[ \frac{dy}{dx} = x(x^2 - 1) \]

• Slope is zero when \( x = -1, 0, \) or \( +1 \) but only stable at \( x = -1 \) and \( +1 \) (bi-stable)
NOTICE THAT THE FREQUENCY OF THE GLASS MOTION CHANGES AS MORE NOISE IS ADDED EVEN THOUGH THE EXCITING FREQUENCY STAYS THE SAME
It has been postulated that organisms such as crayfish are able to “hear” the swimming motions of certain fish by being “tuned” to weak but coherent water motions. Application of stochastic noise aids in this detection by the crayfish.*

As white noise is injected the system may resonate at certain frequencies.

This can cause the original undetectable signal to resonate and produce a higher frequency “peak” as demonstrated below:

Resonance causes peak to be higher
• If the system is bi-stable, a 2\textsuperscript{nd} frequency may replace the first as the highest peak.

• For extremely weak bi-stable signals at the atomic level, noise injection is combined with autocorrelation and extremely long integration times to successfully extract the information about the signal. A SQUID (Superconducting Quantum Interference Device) is sometimes used with liquid helium at 3\textdegree Kelvin.

• Other applications are found in Classical and Quantum Physics and many other disciplines.

• One Biomedical application is in patients who are almost numb in their feet to help them walk.

• Special shoes vibrate just below the level where they can be felt. When the patient walks, the floor can then be felt with the added stimulus.