Lecture I

The Sonar Equation
and
Signal Detection

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What is Sound?

Sound is a mechanical wave motion propagating in an elastic medium. Associated with this wave motion are changes in the local pressure and density.

Sonar, an acronym for SOund NAvigation and Ranging, uses sound energy (generally underwater) to transmit information.
Longitudinal Waves

COMPRESSIONS AND RAREFACTIONS OF LONGITUDINAL WAVES

$\lambda =$ WAVELENGTH
$f =$ FREQUENCY
$c =$ SPEED OF SOUND
Terminology

Psycho/Physiological

Pitch

(Nominal)

Audible Range: 20 – 20,000 Hz

Speech Range: 100 – 4,000 Hz

Infrasonic: Below audible range

Ultrasonic: Above audible range

Physical

Frequency

Intensity

Intensity = $10 \log_{10} \frac{I_1}{I_0} = 20 \log_{10} \frac{P_1}{P_0}$ Decibels (dB)

Waveform

Spectrum, Relative amplitudes/phases

Loudness

Quality
# Nominal Speed of Sound

<table>
<thead>
<tr>
<th>Material</th>
<th>Speed of Sound (m/s)</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum (rolled)</td>
<td>6420</td>
<td>2.7</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>5790</td>
<td>7.9</td>
</tr>
<tr>
<td>Rubber, Gum</td>
<td>1550</td>
<td>0.95</td>
</tr>
<tr>
<td>Fresh Water (25°C)</td>
<td>1498</td>
<td>0.998</td>
</tr>
<tr>
<td>Sea Water (25°C)</td>
<td>1531</td>
<td>1.025</td>
</tr>
<tr>
<td>Air</td>
<td>331</td>
<td>0.0012</td>
</tr>
<tr>
<td>Sea Water</td>
<td>1430→1530</td>
<td></td>
</tr>
</tbody>
</table>
Acoustic Wavelength versus Frequency

Wavelength = sound speed/ frequency

\[ \lambda = \frac{c}{f} \]
Sensitivity of the Human Ear

$0 \text{ dB} = 0.0002 \text{ dyne/sq. cm}$

Percentage of individuals who can hear of various intensities.

Shortly & Williams (53)
Doppler Shift (Passive Sonar)

When a sound source is moving with respect to an acoustic observer, the frequency of the sound is shifted.

\[ f_0 = f_s \frac{c + V_0}{c - V_s} \]

- \( f_0 \) = observed frequency
- \( f_s \) = frequency at the source
- \( c \) = speed of sound
- \( V_0 \) = speed of observer approaching sound source = \( V_1 \cos \Theta \)
- \( V_s \) = speed of source approaching observer = \( V_2 \cos \Theta \)
- \( \Theta \) = bearing angle

Doppler shift is about 0.35 Hz per knot, about 1% at 30 knots
Doppler Shift (Active Sonar)

In active sonar, the approaching target received a signal which is shifted higher in frequency (up-Doppler), then reradiates as a moving source approaching the receiver, causing a doubling of the Doppler shift.

\[
\frac{\Delta f}{f} \approx \frac{2 (V_o + V_s)}{C - (V_o + V_s)} = 6.9 \times 10^{-4}
\]

\[
\Delta f = 0.69 \text{ Hz/(knot)} \text{ (KHz)} \text{ [ACTIVE]}
\]

**REMEMBER**
- Approaching target produces an up-Doppler.
- Receding target produced a down-Doppler.
- Active sonar Doppler shift is double that of passive sonar
  (about 2% at 30 knots)
Lofargram of Simulated Aircraft Overflight

\[ F(t) = \frac{F_{\text{source}}}{\frac{V_0}{c_0} \cos \theta} = \frac{68 \text{ Hz}}{1 - \frac{103}{331} \cos \theta} \]
Decibel (dB)

Decibels are 10 times the logarithm (to base 10) of the ratio of a power or power-related quantity divided by a reference level.

\[ I \text{ (dB re 1 watt/cm}^2\text{)} = 10 \log \frac{I \text{ (watts/cm}^2\text{)}}{I_{\text{ref}}} \]

(watt/cm\(^2\))

Remember the following properties of logarithms:

\[ \log 10^X = X \]

\[ \log [10^X \cdot 10^Y] = \log 10^{(X+Y)} = X + Y \]

To convert from dB to linear units

\[ i \text{ (linear ratio)} = 10^{(0.1)I} \]

Where \( i \) = linear ration normalized by reference unit

\( I = \text{dB value.} \)
Intensity/Pressure/Acoustic Impedance

Intensity is the (sound) power flow per unit area

\[ I = \frac{P^2}{\rho c} \]

where \( P = \) pressure amplitude
\( \rho c = \) acoustic impedance
\( \rho = \) density
\( c = \) speed of sound

Intensity = 10 log \( I/I_{\text{ref}} \)
= 10 log \( (P/P_{\text{ref}})^2 \)
= 20 log \( P/P^2 \)
Pressure Units

- Force per unit area

- MKS units:
  1 Pascal = 1 Newton/m²

- CGS units:
  1 µbar = 1 dyne/cm²

- New reference unit
  1 µPascal = 10⁻⁶ Newton/m²
Power Addition

It is sometimes necessary to sum powers of two numbers which are expressed by dB. The numbers cannot be simply summed in dB form, as that would correspond to multiplication. In order to sum power, first convert from dB to the original units, add, and then express the result in dB. Alternately, the following curve can be used.

![Graph showing the relationship between the difference in dB between two levels and the increment in dB to be added to a higher level.]

Increment in dB (to be added to higher level) vs. Difference in dB between two levels being added.
EXAMPLE FOR THE COMBINATION OF SHIPPING NOISE AND WIND WAVE NOISE

Light Shipping (100 Hz) = 60 dB re 1 μPa

Wind-Wave (100 Hz, SS3) = 63 dB re 1 μPa

Total Noise = 63 + 1.8 dB re 1 μPa

EXAMPLE FOR DETECTION THRESHOLD = -5 dB

$\frac{S}{N}_{\text{REQ}} = -5$ dB

$[(S+N/N)]_{\text{REQ}} = 1.2$ dB
More Examples

If the acoustic pressure in a plane wave is increased by a factor of 3, what is the change in level of the sound?
If the acoustic intensity in a plane wave is increased by a factor of 3, what is the change in level of the sound?
If the acoustic pressure in a plane wave is reduced to 1/4, what is the change in level of the sound?
If the acoustic intensity in a plane wave is reduced to 1/10, what is the change in level of the sound?
Yet More Examples

At a certain location the noise level due to oceanic turbulence is 70 dB, the noise level due to distant shipping is 73 dB, and the self-noise level is 73 dB. What is the total noise level?
Illustration of bathymetrically unimpeded ray paths emanating from Herd Islands with the indicated launch angles. All rays are refracted geodesics along the surface containing the sound channel axis. White boxes indicate locations of oceanographic research stations capable of receiving acoustic signals. (Munk and Forbes, 1989; J. Phys. Oceanography, No. 19, 1965-78; copyright by the American Meteorological Society)
Generalized Shipping Routes
Noise Spectra

General spectrum of deep-sea noise showing five frequency bands of differing spectral slopes; the slopes are given in decibels per octave of frequency (Urick, 1983; Principles of Underwater Sound, 3rd edn;)

Average deep-sea ambient noise spectra (Urick, 1983; Principles of Underwater Sound, 3rd edn;)

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Bathymetry and Sound Speed Structure

Illustration of the conversion of coastal shipping noise, represented by high-angle rays, to noise in the deep sound channel, represented by horizontal rays (adapted from Wagstaff, 1981).

Bathymetric and sound speed structure in the North Pacific Ocean. The noise from distributed shipping sources at high latitudes can enter the sound channel and propagate with little attenuation to lower latitudes. Relationships between the sound speed structure and the prevailing water masses are also illustrated (Kibblewhite et al., 1976).
Ambient Noise Measurements

Northeast Pacific Ocean ambient noise measurements (Morris, 1978): (a) sound speed profile and hydrophone depths; (b) measured noise profiles.
Bathymetric and sound speed structure in the North Pacific Ocean. The noise from distributed shipping sources at high latitudes can enter the sound channel and propagate with little attenuation to lower latitudes. Relationships between the sound speed structure and the prevailing water masses are also illustrated (Kibblewhite et al., 1976).

Ambient Noise Measurements

Directionality of Ambient Noise

The noise field used in the RANDI model (Wagstaff, 1973).

Per-degree horizontal directionality of ambient noise in 10° sectors generated by the RANDI noise model for a frequency of 100 Hz at a depth of 91 m in the North Pacific Ocean (Wagstaff, 1973).
Ambient Noise Near Compact Ice Edge

Variation of median ambient noise sound pressure spectrum levels with distance from a compact ice edge for frequencies of 100, 315 and 1000 Hz in sea state 2 (Diachok, 1980).
Radar Equation Definitions

\[
\begin{align*}
\text{SNR} & = \text{Signal-to-noise ratio} \\
P_T & = \text{Transmitter power} \\
G_T & = \text{Transmitter directional gain} \\
r & = \text{Range to target} \\
\alpha & = \text{Attenuation coefficient} \\
\sigma & = \text{Effective target scattering cross-section} \\
A_R & = \text{Effective area of receiver} \\
KBT & = \text{Thermal noise power} \\
P_{\text{amb}} & = \text{Ambient noise power} \\
P_{\text{rev}} & = \text{Reverberation backscattered power}
\end{align*}
\]

Convert equation to dB by taking 10 log of both sides of the equation.
The Radar/Sonar Equation

\[ \text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} \]

\[ \text{SNR} = \frac{\left( \frac{P_T G_T}{4\pi} \right) \frac{1}{r^2} \cdot \left( \frac{\sigma}{4\pi} \right) \frac{1}{r^2} A_R}{kTB + P_{amb} + P_{rev}} \]

where \( A_R = \frac{\lambda^2}{4\pi} G_{\text{Receiver}} \)

= Gain Against Noise or reverberation
Sonar Equations (dB Values)

Active Sonar – Ambient Noise Background

\[ [SL - 2TL - + TS] = [NL – DT] + DT + SE \]

Active Sonar – Reverberation Background

\[ [SL - 2TL - + TS] = (RL) + DT + SE \]

Passive Sonar

\[ [SL - TL] = (NL – DI) + DT + SE \]

Signal Level = Effective + Required + Signal

Masking Noise to Noise Relative Ratio to DT

Excess Relative to DT
Solving the Sonar Equation

The sonar equation expresses the signal-to-noise ration as function of the sonar and environmental parameters. At the maximum detection range, the received signal power divided by the noise power is equal to the detection threshold. (Signal excess is zero.) Uncertainties in some of the parameters, especially TL and NL, limit the accuracy of prediction to a few dB.
Figure of Merit

FOM equals the maximum allowable one-way transmission loss in passive sonar or the maximum allowable two-way transmission in active sonar.

\[
\text{Passive FOM} = T_L = S_L - [N_L - D_I + D_T]
\]

\[
\text{Active FOM} = 2T_L = S_L + T_S - [N_L - D_I + D_T]
\]

**Note:**
Including TS in active FOM disagrees with Urick, but is a common practice.
FOM is not useful for reverberation limited ranges.
Passive Sonar Prediction

\[ [SL - TL] - [NL - DI] = DT \]

Received Signal Level - Background Masking Noise = Detection Threshold

Urick [75]
Active Sonar Prediction

Urick [75]
## Active Sonar Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Important Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source Level</td>
<td>SL</td>
<td>Input Power</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Conversion Efficiency</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Directivity Index</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cavitation Limitations</td>
</tr>
<tr>
<td>Target Strength</td>
<td>TS</td>
<td>Target Area</td>
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<td></td>
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<td>Target Aspect</td>
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<td>Pulse Duration</td>
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<td>Reverberation Level</td>
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<td>Boundaries</td>
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<td></td>
<td>Scattering Layers</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency</td>
</tr>
</tbody>
</table>
Detection Threshold

\[ DT = 10 \log \frac{\text{Req Signal Power for a Spec. Performance}}{\text{Noise Power per Hz at the Receiver Input}} \]

Detection Threshold (DT) is defined as the ratio (in decibels) of the signal power in the receiver bandwidth to the noise power spectrum level (in a 1 Hz band) measured at the receiver terminals, required for detection at some specified level of correctness of the detection decision [i.e., \( p(D) \) and \( p(FA) \)].

Typical requirements per decision (or per look):

\[ p(D) = 0.5 \text{ or } 0.9 \]
\[ p(FA) = 10^{-6} \]

Urick [75]
Detection Decision

Decide “Signal Present” when the input exceeds the expected value of noise by more than a bias level; otherwise, decide “Signal Absent.”

<table>
<thead>
<tr>
<th>Decision</th>
<th>Signal Present</th>
<th>Signal Absent</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signal Present</td>
<td>Correct DETECTION $p(D)$</td>
<td>False Dismissal $1-p(D)$</td>
</tr>
<tr>
<td>Signal Absent</td>
<td>FALSE Alarm $p(FA)$</td>
<td>Correct Dismissal $1-p(FA)$</td>
</tr>
</tbody>
</table>

Urick [75]
Signal Detection Experiment

GAUSSIAN NOISE GENERATOR
Average noise level $N$
15-second sections

STEADY SINUSOIDAL SIGNALS
Duration $T$
Average Power $S$

THE EXPERIMENT
Signals embedded in half the 15-second noise sections (or echo cycles)
Listeners push button when “signal present”

LISTENER TRAINING
Easily detected signals
Signal levels gradually reduced
Data Collection

• “HIT” SCORED WHEN LISTENER RESPONDED WITHIN 1 SEC. AFTER SIGNAL OCCURRENCE

• “FALSE ALARM” SCORED WHEN LISTENER RESPONDED TO NOISE

• LISTENERS WERE NOT RESTRICTED TO ONE RESPONSE PER NOISE SECTION

• SCORES WERE TABULATED USING LAST RESPONSE IN EACH SECTION
Estimates of $P_D$ and $P_{FA}$

\[
P_D = \frac{\text{NUMBER OF "HITS"}}{\text{NUMBER OF NOISE SECTIONS WITH SIGNALS}}
\]

\[
P_{FA} = \frac{\text{NUMBER OF "EMPTY SECTIONS" IN WHICH RESPONSES WERE MADE}}{\text{NUMBER OF "EMPTY" NOISE SECTIONS}}
\]

“EMPTY” NOISE SECTION IS NOISE SECTION WITH NO EMBEDDED SIGNAL.
Experiment Results

♦ LISTENED TO SEVERAL HUNDRED NOISE SECTIONS

• Different days
• Different instructions on strictness about responding

♦ $P_D$ PLOT AGAINST $P_{FA}$ GROUPED AROUND THEORETICALLY COMPUTED $\frac{S}{N} = 0$ dB Curve

$\frac{S}{N} (\text{dB}) = 10 \log_{10} \left[ \frac{\text{Average Signal Power (watts)}}{\text{Average Noise Power (watts)}} \right]$  

♦ WHEN AVERAGE S/N CHANGED TO $\pm 2$ dB, MEASUREMENTS CLUSTERED AROUND S/N = $\pm 2$dB CURVES, ETC.

NOTE: $\log_{10}(1) = 0$ means $<S> = <N>$
Receiving Operating Characteristics (ROC) Operating Curves

D, DETECTION PROBABILITY

F, FALSE ALARM PROBABILITY

o Observer 1
x Observer 2
△ Observer 3

S/N = 2 DB
S/N = 0
S/N = -2
S/N = 0

EYE

EAR
Interpretation of Results

- $P_D$ increases with S/N
- $P_{FA}$ decreases as S/N increases
- Visual detection
  - Noise sections written out as pen recordings (such as time bearing plots)
  - Observers detect signal by viewing recorder plots
  - Different set of $P_D$ vs. $P_{FA}$ curves
  - Eye not as good (for detection) as ead
Concept Leading to Construction of ROC Curves

Output Levels:

- Energy detector – Mean square amplitude of power
- Voltmeter
  - Oscillate about (B) when no signal present
  - Oscillate about (A) when signal present
Gaussian Distribution (Model of Random Noise)

GAUSSIAN PROBABILITY DENSITY FUNCTION:

\[ n(x) = \frac{1}{\sqrt{2\pi} \sigma} \cdot \frac{-(x-\mu)^2}{e^{2\sigma^2}} \]

\[ y = \frac{x-\mu}{\sigma} \]

\[ n(y) = \frac{1}{\sqrt{2\pi}} \cdot \frac{-y^2}{e^2} \]

\[ \mu \pm \sigma : 68\% \]
\[ \mu \pm 2\sigma : 95\% \]
\[ \mu \pm 3\sigma : 99.7\% \]

\[ \mu = \text{mean} \]
\[ \sigma = \text{standard deviation} \]

Central Limit Theorem: The random selection of values from any distribution (Gaussian or not) tends to a Gaussian distribution with the same mean and standard deviation as was in the original distribution.
Normal Distribution Probability Density and Cumulative Distribution Functions

Normal Distribution Density

\[ \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) \]

Cumulative Distribution Function

\[ \frac{1}{2} \left( 1 + \text{erf} \frac{x - \mu}{\sigma \sqrt{2}} \right) \]
Normal Bell-Shaped Curve

Percentage of cases in 8 portions of the curve:

-4σ: .13%
-3σ: 2.14%
-2σ: 13.59%
-1σ: 34.13%
0: 34.13%
+1σ: 13.59%
+2σ: 2.14%
+3σ: .13%

Standard Deviations:

Cumulative Percentages:
0.1% 2.3% 15.9% 50% 84.1% 97.7% 99.9%

Cumulative Percentiles:
1 5 10 20 30 40 50 60 70 80 90 95 99

Z scores:
-4.0 -3.0 -2.0 -1.0 0 1.0 2.0 3.0 4.0

T scores:
20 30 40 50 60 70 80

Standard Nine (Stanines):
1 2 3 4 5 6 7 8 9

Percentage in Stanine:
4% 7% 12% 17% 20% 17% 12% 7% 4%
Computing ROC Curves

**DETECTABILITY INDEX:**

\[ d'e = \frac{\mu_{S+N} - \mu_N}{\sigma} = \frac{y_0}{\sigma} \]

- \( P(y/N) = \) Prob. Density of output \( y \), given \( N \) as input
- \( P(y/SN) = \) Prob. Density of output \( y \), given \( S+N \) as input

**IDEAL ENERGY DETECTOR**

\[ d'e = \frac{E}{N_0} \cdot \frac{1}{(WT)^{1/2}} = \frac{S}{N} \cdot (WT)^{1/2} \]

- \( E = \) Total signal energy received during time \( T \) in band \( W \)
- \( S = \) Signal power in band \( W \) (note: \( E = ST \))
- \( T = \) Sample time of detector
- \( W = \) Input bandwidth of the detector system
Evaluation of $P(D)$ and $P(F)$

$$P(FA) = \int_{-\infty}^{\infty} P_N(Y) \, dY = \int_{-\infty}^{\infty} (\text{Prob. density of noise})$$

$$P(D) = \int_{-\infty}^{\infty} P_{SN}(Y) \, dY = \int_{-\infty}^{\infty} (\text{Prob. density of signal and noise})$$
ROC Curves for an Energy Detector

ROC CURVES FOR AN ENERGY DETECTOR. P(D) IS DETECTION PROBABILITY, P(FA) IS FALSE ALARM PROBABILITY, \( d' \) IS THE DETECTABILITY INDEX, S AND N ARE INPUT SIGNAL AND NOISE POWER IN THE DETECTOR BANDWIDTH W, T IS THE SAMPLE TIME OF THE DETECTOR.

\[ d = (d'_{e})^2 \sim \text{Output S/N} \]
Required signal-to-noise (visibility factor) at the input terminals of a linear-rectifier detector as a function of probability of detection for a single pulse, with the false-alarm probability ($P_{FA}$) as a parameter, calculated for a non-fluctuating signal [Blake 69]