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Satellite Constellation Geometry

by Robert A. Nelson

Satellite constellation geometry has been studied as a theoretical problem since the early 1960s. The analysis originally had limited application to photographic reconnaissance and Earth resource missions. However, at present it has achieved particular relevance for the numerous satellite systems under development that offer a variety of new services, including mobile telephony, message and data transfer, and position determination. The problem combines the physics of orbits with optimization of the orbit geometry so as to provide the required Earth coverage while minimizing the number of satellites.

Notable contributions to the theory of constellation geometry have been made by Walker, Draim, Ballard, and Adams and Rider. Walker made an extensive study and found that at least five satellites are required for continuous global coverage from circular orbits at a common altitude and inclination. His method of classifying constellation types with the notation $T/P/F$ is frequently used, where $T$ is the total number of satellites, $P$ is the number of evenly spaced orbital planes and $F$ determines the phase spacing between adjacent planes. Draim found that continuous coverage could be attained by only four satellites in elliptical orbits. Ballard also studied the optimization of satellites in inclined circular orbits, which he called “rosette constellations,” using a satellite triad approach. This method minimizes the largest distance between the observation point and any subsatellite point. Adams and Rider deduced the optimum configurations for polar orbit constellations for single or multiple satellite levels of coverage over the entire Earth or above a specified latitude, using a street-of-coverage approach. This method considers a ground swath that is continuously covered.

**ALTITUDE**

The altitude of the satellite orbit is the primary characteristic of the satellite constellation. It is chosen on the basis of both physical and geometric considerations, including signal propagation delay, signal power, avoidance of the Van Allen radiation belts, time of satellite visibility and coverage area.

The altitude regimes have been divided by convention into Low Earth Orbit (LEO), Medium Earth Orbit (MEO) and Geostationary Orbit (GEO). The altitude of LEO is roughly between 500 km and 1,500 km. The lower bound is chosen to avoid excessive stationkeeping due to residual atmospheric drag. The upper bound is chosen so as to lie below the first Van Allen radiation belt. The altitude of MEO can be taken to be approximately within the range 5,000 km to 15,000 km so as to be within the first and second Van Allen belts. The limits are ten times those of LEO. The altitude of GEO is uniquely 35,786 km in the equatorial plane. At this altitude the period of revolution is exactly equal to the period of rotation of the Earth (23.934 h), so that a satellite appears to remain over a fixed point on the equator. A fourth orbit category is the highly elliptical orbit (HEO), in which the apogee may be beyond the geostationary orbit.

The two principal factors that have created interest in LEO and MEO for satellite communications are the low signal propagation delay and the limitations on gain and power of the Earth terminal. The round trip signal delay for a two-way conversation via satellite at an altitude of 10,000 km is 130 ms, and for a satellite at an altitude of 1,000 km it is only 13 ms. In contrast, the propagation delay from GEO for a two-way conversation is over half a second, which is distracting at best and can be intolerable for digital data transmission using error correcting protocols that require retransmission of blocks with detected errors.

Handheld telephones by their nature must have low gain (on the order of 1 dB) because they must be omnidirectional and have fixed power limits (on the order of 350 mW) to safeguard human health. The Earth terminal gain and power determine the required size of the satellite antenna, which must be large enough to provide sufficient link margin. Also, the bandwidth available is limited, so the total coverage area is usually divided into a cellular pattern of spot beams to permit frequency reuse. The cell size is determined by the size of the antenna and the orbit altitude. As the orbit gets higher, it is necessary to use a larger antenna on the spacecraft to achieve a given spot size on the Earth. For example, at L-band (1615 MHz), a 17 meter spacecraft antenna in GEO would be required for the same cell size as a 0.5 meter antenna in LEO. Thus, LEO and MEO are preferable to GEO for mobile hand-held telephony.

Other considerations that affect the choice of altitude are satellite visibility and eclipse time. At Low Earth Orbit the period of revolution is approximately 100 minutes. For a typical pass, the satellite is visible for only about ten minutes. Thus, frequent handover is required for mobile telephony. In addition, during times of the year when the orbital plane is parallel to the direction to the Sun, the satellite is eclipsed for about 30 minutes, or about one third of the orbital period. Consequently, there is a significant demand on battery power, with up to 5,000 charge/discharge cycles per year in Low Earth Orbit. With present nickel-hydrogen battery technology, a battery rated for 10 to 15 years in GEO would have a life of about 5 years in LEO. On the other hand, in Medium Earth Orbit the orbital period is six to eight hours and the time of visibility of a single satellite is over an hour. There are fewer eclipse cycles and battery lifetime is typically seven years.

**INCLINATION**

The second fundamental parameter of a satellite constellation is its orbital inclination. The choice is governed by the global coverage requirement, the level of coverage, and the minimum angle of elevation. Inclinations of direct circular orbits are generally around 50°. This inclination permits coverage of temperate zones and populated regions of the Earth, while allowing more than one satellite to be visible from a given point for reasonable constellation sizes. Polar constellations have inclinations near 90°, which permits global coverage with the fewest satellites. Retrograde orbits (such
as Sun-synchronous orbits) have inclinations greater than 90°.

A great advantage of inclined or polar LEO and MEO constellations is that they afford high angles of elevation. Elevation angles of from 20° to 40° may be required to avoid blockage from tall buildings in urban areas. These angles are not possible from GEO, even at moderate latitudes of 45°. Many of the capitals of Europe, including Paris, London, Berlin, Warsaw and Moscow, are north of this latitude. Furthermore, a geostationary satellite is below the horizon if the latitude is greater than 81°.

**ECCENTRICITY**

The third important orbital parameter is the eccentricity, which determines the orbit’s shape. For circular orbits, the eccentricity is zero and the satellite moves at uniform speed. For elliptical orbits, however, the eccentricity is between 0 and 1. The satellite moves fastest at perigee, or the point closest to the Earth, and slowest at apogee, or the point farthest from the Earth. By adjusting the position of the apogee, the dwell time of the satellite can be maximized over the region of interest.

Earth oblateness perturbations restrict the inclination of elliptical orbits to 63.4° or 116.6° for satellite communications. These are the only two inclinations at which the major axis remains fixed, so that the apogee remains over the specified latitude. At all other inclinations the gravitational harmonics of the Earth due to its oblate shape cause the major axis to rotate. For example, the Russian 12 hour Molniya orbit is a highly elliptical orbit inclined at 63.4°. The perigee altitude is 1,006 km and the apogee altitude is 39,362 km with apogee over the northern hemisphere. A Molniya satellite spends nearly 11 hours over the northern hemisphere and only 1 hour over the southern hemisphere per revolution.

**CONSTELLATION CONFIGURATION**

The configuration of the constellation is defined by the number of orbital planes $p$ and the number of satellites per plane $s$. The values of $p$ and $s$ should be chosen so as to minimize the total number of satellites $N$ that are required to provide the specified level of coverage, where $N = p \cdot s$.

For a given minimum angle of elevation $\theta$, the angle $\gamma$ with respect to the Earth’s center between the subsatellite point and edge of coverage is given by

$$\gamma = \arccos \left( \frac{\cos \theta}{1 + h/R_E} \right) - \theta$$

where $h$ is the satellite altitude and $R_E$ is the radius of the Earth (6,378 km). The total coverage area may be estimated from the formula

$$S = 2\pi R_E^2 (1 - \cos \gamma)$$

Ideally, $S$ should be as large as possible, but it is usually subdivided into an array of cells to permit frequency reuse. It may be limited by the required satellite antenna gain, which is approximately given by $G = 4\pi h^2 / (n \cdot S)$, where $n$ is the number of cells. The diameter $D$ of the antenna is then given by $G = \eta (\pi D / \lambda)^2$, where $\lambda$ is the wavelength and $\eta$ is the efficiency. These relations imply that the antenna diameter is proportional to the altitude for a given cell size.

The coverage geometry problem is simplest for polar constellations. For global coverage with optimum phasing, the point of intersection of overlapping circles of coverage in one plane coincides with the boundary of a circle of coverage in a neighboring plane. Satellites in adjacent planes revolve in the same direction. However, there is a “seam” in the constellation pattern between the first and last planes, where the satellites revolve in opposite directions. For a given number of planes $p$ and number of satellites per plane $s$, the Earth central angle $\gamma$ and ground swath half-width $\Gamma$ are determined by the equations

$$\cos \Gamma = \frac{\cos \gamma}{\cos (\pi / s)}$$

and

$$(p - 1) \alpha + \beta = \pi$$

where $\alpha = \Gamma + \gamma$ is the spacing between co-rotating planes and $\beta = 2 \Gamma$ is the spacing between counter-rotating planes.

For example, the original Iridium constellation, based on a paper by Adams and Rider, consisted of 77 satellites distributed into seven planes with 11 satellites per plane. The constellation was named after the element iridium, whose atomic structure consists of 77 electrons orbiting the nucleus.) Therefore, the Earth central angle $\gamma$ was 18.5° and the ground swath half-width $\Gamma$ was 8.6°. Also, $\alpha$ was 27.1° and $\beta$ was 17.2°. For a minimum elevation of 10° at edge of coverage, the corresponding altitude was 765 km. This altitude satisfied the constraints that it was sufficiently high that atmospheric drag was negligible, it was sufficiently low that it avoided the Van Allen radiation environment, and the cost of satellite deployment was moderate.

The Iridium constellation has been revised by the elimination of one plane to reduce the number of satellites. It now consists of 66 satellites distributed into six planes with 11 satellites per plane at an altitude of 780 km. The plane separation is 31.6° and the orbital inclination has been changed to 86° as a precaution against collisions at the poles. The angle of elevation at edge of coverage on the equator is 8.2°.

The geometric problem for inclined constellations is somewhat more complicated. Walker, Ballard and Rider have examined this problem using a variety of assumptions and techniques. For example, the Globalstar constellation consists of 48 satellites, with 6 satellites in each of 8 orbital planes, at an altitude of 1406 km and inclined at 52°. This constellation was based on the Walker 48/8/1 “delta” pattern and was refined by computer modeling. A basic requirement of this system is that two satellites must be visible from any point. The communications link uses code division multiple access (CDMA) with path diversity. Each mobile telephone receives a signal from each of two satellites at half power to minimize blockage and multipath effects.

**EARTH OBLATENESS**

Earth oblateness has two important effects on a orbit. First, as mentioned previously, it causes the major axis to rotate. The rate of change of the perigee angle is

$$\frac{d\omega}{dt} = \frac{4.982}{(1 - e^2)^{3/2}} \left( \frac{R_E}{a} \right)^{3.5} (5 \cos^2 i - 1)$$

expressed in degrees per day, where $a$ is the semimajor axis, $e$ is the eccentricity and $i$ is the inclination. This equation...
implies that the major axis is stable only for inclinations of 63.4° and 116.6°, which are the only angles that make the right hand side of the equation equal to zero.

Oblateness also causes the ascending node of the orbit to drift. The rate of drift is given by the formula

\[
\frac{d\Omega}{dt} = -\frac{9.964}{(1-e^2)^{3/2}} \left( \frac{R_E}{a} \right)^{3.5} \cos i
\]

expressed in degrees per day. For inclinations less than 90° the ascending node drifts westward, while for inclinations greater than 90° the ascending node drifts eastward. The ascending node does not drift for polar constellations, for which the inclination is 90°. For example, for the Globalstar constellation, the ascending node drifts westward at the rate of 3° per day.

The operational impact of ascending node drift on LEO constellations with intermediate inclinations is the penalty on stationkeeping fuel. In principle, if all the satellites had identical circular orbit altitudes and inclinations, the orbit planes would drift in unison and the relative geometry would remain constant. However, in practice, there are inevitable orbit insertion errors during deployment. In the preceding example, the difference in ascending nodes would accumulate to 0.5° in one year for each kilometer of error in altitude and would accumulate to 2.5° in one year for each 0.1° of error in inclination, compared to the nominal orbit.

SUN-SYNCHRONOUS ORBITS

The drift in ascending node has one important practical application. If the altitude, inclination and eccentricity are chosen so that the ascending node drifts eastward at the same rate as the Earth revolves around the Sun (0.9856° per day), then the Earth–Sun line would maintain a constant orientation with respect to the orbital plane. This type of orbit was first used by the Landsat satellites for Earth photography missions. Landsat-1 was launched in July 1972 into a 910 km altitude orbit inclined at 99°.

If the orbital plane is initially oriented perpendicular to the direction of the Sun, the satellite will always remain illuminated. The solar array would not require a tracking mechanism and batteries would be needed only for contingencies. Another advantage of Sun-synchronous orbits is that the orbital period can be synchronous with the mean solar day instead of the sidereal day over a given point on Earth, so that the satellite maintains the same time-of-day schedule.

The E-Sat satellite system provides an example based on these considerations. This system will provide data messaging and data retrieval services for public utilities and petroleum companies, direct-to-home television broadcast services and the financial services industry. The satellite orbit, is a Sun-synchronous circular orbit with a period of revolution that is a submultiple of a mean solar day. It has thus been given the name of “doubly-synchronous orbit.” Since the orbit is Sun-synchronous, the satellite maintains the same time-of-day schedule.

The orbital plane is to be oriented perpendicular to the Earth-Sun line and the satellite solar array will be constantly illuminated. The ground trace will repeat itself every day. It was also required that the altitude must be within the range 1,000 km < h < 1,500 km so that the atmospheric drag would be negligible and would not impinge on the first Van Allen radiation belt. Therefore, a circular orbit with an altitude of 1,262 km and an inclination of 100.7° was chosen.

For a minimum elevation angle of 20°, the Earth central angle γ is 18.3°. The corresponding coverage area is 13 million square kilometers, or roughly the size of CONUS. This coverage area implies that the maximum satellite gain must be 1.9 dB at 149.5 MHz. Therefore, for the given Earth terminal power and gain, method of modulation and coding, and various losses, the maximum data rate that can be supported by the communications link is determined. Three satellites will be deployed into one plane to meet the required capacity of the anticipated market. An additional three satellites may be added at a later time. If the latter satellites are deployed into a different orbit plane, they will have the required modifications to the electrical power subsystem to permit solar tracking and accommodate eclipse periods.

A satellite orbit based on similar considerations was proposed in a 1984 NASA-Lewis study for the Voice of America as one of several concepts for a direct broadcast satellite system. In this case, elliptical Sun-synchronous orbits with an integral number of revolutions per mean solar day were investigated. An elliptical orbit was considered because it would provide a long dwell time over the region to be covered with proper positioning of the apogee. Since the major axis could not rotate and since the inclination of a Sun-synchronous orbit must be greater than 90°, the inclination of 116.6° was required. With this additional level of synchronism, the orbit was given the name “triply-synchronous orbit.” The only orbit with an integral number of revolutions per day that does not intersect the Earth is the three hour orbit, with a perigee altitude of 521 km and an apogee altitude of 7,843 km. An identical orbit concept has been adopted by Mobile Communications Holdings, Inc. (MCHI) for their “Borealis orbit” of its proposed Ellipsat constellation.

NAVIGATION SATELLITES

The Global Positioning System (GPS) is a fully operational satellite system for high precision position determination developed by the U.S. Department of Defense. The GPS constellation consists of 21 operational satellites and three in-orbit spares in circular orbits at an altitude of 20,182 km. The orbital period is one-half a sidereal day, or 11.967 hours. The ground track repeats itself every two revolutions, with the result that a given satellite appears over the same point 4.1 minutes earlier than the previous day. Four satellites are deployed into each of six orbital planes inclined at 55°. At least four satellites are visible at all times from any point on Earth.

Each satellite carries two cesium and two rubidium atomic clocks that maintain a highly stable time and frequency reference. The satellite orbit and clock information is transmitted on each of two L-band carriers (1575.42 MHz and 1227.60 MHz). Two frequencies are used to measure and compensate for the effect of ionosphere and troposphere delay. The baseband signal is modulated by two spread-spectrum pseudorandom noise codes: a precision (P) code at 10.23 Mbps for military use that repeats every 38 weeks and a clear access (C/A) code at 1.023 Mbps for satellite acquisition and civilian use that repeats every 1 ms. Different satellites use different portions
of the same P code. The user's receiver generates an identical code and measures the distance to the satellite by means of an autocorrelation circuit that determines the phase difference needed to align the two codes. The simultaneous measurement of PRN signals from four satellites permits a three-dimensional determination of position with a resolution of better than 10 meters using the P code or between 100 meters and 300 meters with the C/A code. GPS satellites are also used for time comparison between standards laboratories by common view measurements with a precision of a few nanoseconds.

The Russian Global Orbital Navigation Satellite System (GLONASS) is a similar system under development, consisting of 24 satellites at an altitude of 19,132 km evenly distributed into three orbital planes inclined at 64.8°. The orbital period is 11.263 hours, so the ground track repeats itself every eight days. In contrast to GPS, which uses only two frequencies for the entire system, each GLONASS satellite is assigned its own two frequencies within the bands 1240 - 1260 MHz and 1597 - 1617 MHz. Satellites are distinguished by radio-frequency channel instead of by pseudorandom noise code. A single code is used, repeating every 1 ms.

**CONCLUSION**

The basic principles of satellite constellation design have been reviewed and several actual examples have been described. These examples illustrate how various design considerations lead to the choice of orbit, which then drives the choice of link parameters to meet the system requirements.