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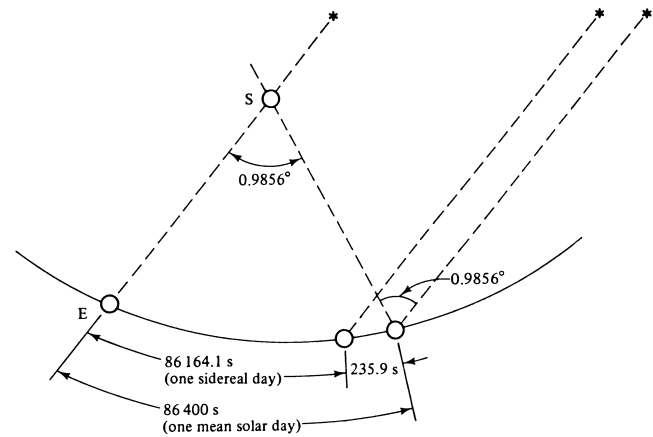
What Is the Radius of the Geostationary Orbit?

by Robert A. Nelson

Most communications satellites operate from the geostationary orbit, since from this orbit a satellite appears to hover over one point on the equator. An Earth station antenna can therefore be pointed at a satellite in a fixed direction and tracking of the satellite across the sky is not required. The basic question to be discussed is, "What is the radius of the geostationary orbit?"

The geostationary orbit must satisfy three conditions: (1) the velocity must be in the direction and sense of the Earth's rotation; (2) the velocity must be constant; and (3) the period of revolution must exactly match the period of rotation of the Earth in inertial space. The first condition implies that the orbit must be a direct orbit in the equatorial plane. The second condition implies that the orbit must be circular. To satisfy the third condition, the radius of the orbit must be chosen to correspond to the required period given by Kepler's third law. According to this law, the square of the orbital period is proportional to the cube of the semimajor axis.¹

The problem reduces to determining the value of the orbital period. However, it is not simply 24 hours, or one mean solar day. The mean solar day is equal to the average time interval between successive transits of the Sun over a given meridian and is influenced by both the rotation of the Earth on its axis and the motion of the Earth along its orbit. Instead, the appropriate period of the geostationary orbit is the sidereal day, which is the period of rotation of the Earth with respect to the stars. One sidereal day is equal to 23 h 56 m 4.0905 s of mean solar time, or 86 164.0905 mean solar seconds. Using this value in Kepler's third law, we compute the orbital radius as 42 164.172 km.



Relationship between the sidereal day and the mean solar day.

Yet even this value for the orbital period is not quite correct because the Earth's axis precesses slowly, causing the background of stars to appear to rotate with respect to the celestial reference system. The Earth's axis is tilted by 23.4° with respect to a line perpendicular to the orbital plane and executes a conical motion with a precessional period of about 26 000 years. Therefore, the sidereal day is less than the true period of the Earth's rotation in inertial space by 0.0084 seconds. On this account, the period of the geostationary orbit should be 86 164.0989 mean solar seconds. The corresponding orbital radius is 42 164.174 km.

There is also a correction due to the unit of time itself. The mean solar second is defined as 1/86 400 of a mean solar day. However, in terms of the second of the International System of Units (SI), defined by the hyperfine transition of the cesium atom, the present length of the mean solar day is about 86 400.0025 seconds. The mean solar day exceeds a day of exactly 86 400 seconds by about 2.5 milliseconds due to slowing of the Earth's rotation caused by the Moon's tidal forces on the shallow seas. This extra time accumulates to nearly one second in a year and is compensated by the occasional insertion of a "leap second" into the atomic time scale of Coordinated Universal Time (UTC). Adding this increment to the orbital period, we obtain 86 164.1014 seconds. The corresponding orbital radius is 42 164.175 km.

The analysis so far has assumed that the Earth can be regarded as a perfect sphere. However, in reality the Earth's shape is more nearly oblate. The equatorial radius is 6378.137 km, while

the polar radius is 6356.752 km. The gravitational perturbation due to oblateness causes the radius to be increased by 0.522 km.² The resulting geostationary orbital radius is 42 164.697 km.

In practice, once the satellite is operational in the geostationary orbit, it is affected by a variety of perturbations that must be compensated by frequent stationkeeping maneuvers using thrusters onboard the spacecraft. These perturbations are caused by the gravitational attractions of the Sun and the Moon, the slightly elliptical shape of the Earth's equator, and solar radiation pressure. Because the orbit is constantly changing, it is not meaningful to define the orbit radius too precisely. By comparison, using recent data for 16 Intelsat satellites, we obtain a semimajor axis with a mean of 42 164.80 km and a standard deviation of 0.46 km.

A perfectly geostationary orbit is a mathematical idealization. Only the distinction between the mean solar day and the sidereal day needs to be taken into account. Therefore, it is customary to quote a nominal orbital period of 86 164 seconds and a radius of 42 164 km. The height above the equator is 35 786 km and the orbital velocity is 3.075 km/s.

¹ Mathematically, Kepler's third law may be expressed as $T^2 = (4 \pi^2 / GM) a^3$, where T is the period, a is the semimajor axis, and GM is the gravitational constant for the Earth, whose value is 398 600.5 km³/s². For a circular orbit, the semimajor axis a is equal to the radius r .

² The correction is $\Delta r = \frac{1}{2} J_2 (R_E / r)^2 r$, where r is the orbital radius, R_E is the Earth's radius, and J_2 is the Earth's oblateness coefficient, 0.001 083.