Professional Development Short Course On:
Radar Systems Analysis & Design using MATLAB

Instructor:

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The Radar Equation

- Consider a radar with an omni directional antenna. The peak power density (power per unit area) at any point in space is

\[ PD = \frac{P_t}{4\pi R^2} \]

where

- \( P_t \): Peak Transmitted Power
- \( 4\pi R^2 \): Area of a Sphere of Radius \( R \)

- The power density at a given range away from the radar (assuming a lossless propagation medium) is
Radar systems utilize directional antennas in order to increase the power density in a certain direction.

Directional antennas are usually characterized by the antenna gain and the antenna effective aperture, which are related by

\[ G = \frac{4\pi A_e}{\lambda^2} \]

- \( G \) = Antenna Gain
- \( A_e \) = Effective Aperture (\( m^2 \))
- \( \lambda \) = Wavelength (\( m \))
The Radar Equation

- The antenna gain is also related to the azimuth and elevation beam widths by

\[ G = k \frac{4\pi}{\theta_a \theta_e} \quad k \leq 1 \]

- Azimuth Beamwidth (radians) \( \theta_a \)
- Elevation Beamwidth (radians) \( \theta_e \)

- An accurate approximation for antenna gain is

\[ G = \frac{26000}{\theta_e \theta_a} \quad \theta_e \theta_a \text{ (degrees)} \]

- The power density at a given range from the radar using a directive antenna is then given by

\[ \frac{P_d}{P_t} = \frac{G}{4\pi R^2} \]
Antenna Beamwidth

-3 dB from peak

θ = 8.75°
The Radar Equation

- When the radiated energy from a radar impinges on a target, the induced surface currents on that target re-radiate or scatter electromagnetic energy in all directions.
The amount of the scattered energy is proportional to the target size, orientation, physical shape, and material, which are all lumped together in one target-specific parameter called the Radar Cross Section (RCS) and is denoted by $\sigma$.

The radar cross section is defined as the ratio of the power reflected back to the radar to the power density incident on the target.

$$\sigma = \frac{P_r}{P_D} \quad (m^2)$$

- $P_r$ = Reflected Power
- $P_D$ = Incident Power Density
Signature Modeling Techniques

There are different ways to solve the EM scattering problem. The method used depends on variables such as:

- Desired accuracy and appropriateness of the method
- Number of CPUs and amount of RAM available
- Amount of wall time available

The different methods are often grouped together into what are called “low” and “high” frequency computational techniques.

“Low frequency” methods solve the scattering problem in an “exact” sense, incorporating all electromagnetic effects. They are often limited by problem size or computer power.

“High Frequency” methods approximate the scattering problem. They are generally less accurate, though often much faster and less demanding on CPU resources than a low frequency method.
Finite Difference Time Domain (FDTD)

- FDTD solves the time-domain version of Maxwell's Equations. FDTD is considered to be a “full wave” or “exact” method.
- For scattering problems, the target and some adjoining space must be discretized on a rectangular grid to support the electromagnetic fields.
- FDTD has high memory and CPU demands, and is not typically used for targets that are electrically large.

\[
\begin{align*}
\mu \frac{\partial H_x}{\partial t} &= \frac{\partial E_y}{\partial y} - \frac{\partial E_z}{\partial z} \\
\mu \frac{\partial H_y}{\partial t} &= \frac{\partial E_x}{\partial x} - \frac{\partial E_z}{\partial z} \\
\mu \frac{\partial H_z}{\partial t} &= \frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y}
\end{align*}
\]

\[
\begin{align*}
\epsilon \frac{\partial E_x}{\partial t} &= \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \vec{J}_x \\
\epsilon \frac{\partial E_y}{\partial t} &= \frac{\partial H_x}{\partial y} - \frac{\partial H_z}{\partial x} - \vec{J}_y \\
\epsilon \frac{\partial E_z}{\partial t} &= \frac{\partial H_x}{\partial x} - \frac{\partial H_y}{\partial y}
\end{align*}
\]

3D Maxwell Equations  
FDTD Grid  
Time-Domain Field
Method of Moments (MoM)

- The MoM is used in the scattering problem in the frequency-domain by solving the integral Maxwell’s Equations. It is considered to be a “full wave” or “exact” method.
- The MoM solves for the induced surface current on an object by discretizing the target surface into a number of subdomain “basis functions”. The number of basis functions (N) needed is proportional to the radar wavelength.
- The MoM creates a matrix equation of order $N^2$. This ultimately limits its application to problems of small electrical size.

\[
\vec{E}_s = -j\omega \vec{A} - \frac{j}{\epsilon \mu \omega} \nabla \nabla \cdot \vec{A}
\]

\[
4\pi \hat{n} \times \vec{H} = 2\pi \vec{J} + \hat{n} \times \iint_S \vec{J} \times \frac{e^{jkR}}{R} \, dS
\]
The Fast Multipole Method (FMM/MLFMA) can be used to reduce the complexity of the MoM matrix system and allow the MoM to be used in problems previously unsolvable.

Reliable, industrial-strength FMM software is slowly becoming commercially available. These codes still require supercomputer-class hardware to solve very large MoM problems.

8 Wavelength Sphere

81920 unknowns

Regular MoM:

60 GB RAM

FMM:

1.2 GB RAM

A factor of 40/1!
Approximate (High-Frequency) Methods

- “High Frequency” methods approach the scattering problem by assuming the target is very large compared to the radar wavelength.

- Approximations can then be made to simplify and expedite signature prediction by considering different scattering mechanisms separately.

- Physical Optics (PO) is a commonly used method that approximates the surface currents by assuming the surface is locally flat. This method does well at specular but is fair to poor at off-specular angles.

- The Physical Theory of Diffraction (PTD) supplements PO by accounting for single diffractions from edges.

- Shooting and Bouncing Rays (SBR) supplements PO and PTD by using ray-optics to model multiple-bounce scattering.

- Other scattering mechanisms (traveling and creeping waves) can be considered, but are difficult to formulate analytically for general targets.
Comparison of MoM and Approximate Methods

- **Target:** 16 x 8 inch “Top Hat”
- **Monostatic RCS Calculated at 7 GHz**
- **Target approximately 10 x 5 wavelengths in electrical size.**

![Graphs showing comparison of PO Only, PO + PTD, and PO + PTD + SBR](image)
Signature Modeling Software

- **Finite Difference Time Domain (FDTD)**
  - Remcom *xfdtd* (packaging, machines)
  - EMS+ *EZ-FDTD* (packaging, machines)

- **Finite Element Method (FEM)**
  - Ansys, Inc. *ANSYS* (packaging, machines)

- **Method of Moments (MoM)**
  - EM Software and Systems *FEKO* (wire and planar antennas)
  - Tripoint Industries *Serenity* (Scattering by MoM/MLFMA)
  - Tripoint Industries *Galaxy* (Scattering by MoM-BoR)

- **Physical Optics / PTD / SBR**
  - Tripoint Industries *lucernhammer MT* (High-frequency scattering)
  - SAIC *xPatch* (High-frequency scattering)
## The Radar Equation

### Typical RCS values of various objects at X-Band

<table>
<thead>
<tr>
<th>Object</th>
<th>RCS (m²)</th>
<th>RCS (dBsm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pickup Truck</td>
<td>200</td>
<td>23</td>
</tr>
<tr>
<td>Automobile</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>Jumbo Jet</td>
<td>400</td>
<td>20</td>
</tr>
<tr>
<td>Commercial</td>
<td>40</td>
<td>16</td>
</tr>
<tr>
<td>Cabin Cruiser Boat</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Large Fighter Aircraft</td>
<td>6</td>
<td>7.78</td>
</tr>
<tr>
<td>Small Fighter Aircraft</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Adult Male</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Conventional Winged Missile</td>
<td>0.5</td>
<td>-3</td>
</tr>
<tr>
<td>Bird</td>
<td>0.01</td>
<td>-20</td>
</tr>
<tr>
<td>Insect</td>
<td>1x10⁻⁵</td>
<td>-50</td>
</tr>
<tr>
<td>Advanced Tactical Fighter</td>
<td>1x10⁻⁶</td>
<td>-60</td>
</tr>
</tbody>
</table>
The Radar Equation

- The power scattered by the target is then
  \[ P_{\text{target}} = \frac{PG\sigma}{4\pi R^2} \]  (watts)

- Recalling that the power density at a given range is
  \[ P_r = \frac{P_t}{4\pi R^2} \]

- Substituting the target's scattered power gives the total power density delivered back to the antenna
  \[ P_{\text{antenna}} = \frac{P_t G_{\text{target}}}{\left(4\pi R^2\right)^2} \]
The Radar Equation

- Multiplying by the effective area of the antenna gives the total power received by the radar.

\[ P_{\text{radar}} = \frac{P_t G \sigma}{(4\pi R^2)^2} \cdot A_e \]

- Substituting for the effective aperture then gives

\[ P_{\text{radar}} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \text{ (watts)} \]
The Radar Equation

Denote the minimum detectable signal power by \( S_{\text{min}} \).

It follows that

\[
S_{\text{min}} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^3}
\]

Rearranging the equation gives the maximum Range to the target:

\[
R_{\text{max}} = \left( \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\text{min}}} \right)^{1/4} \text{ (meters)}
\]
The Radar Equation

- In realistic situations, the returned signals received by the radar will be corrupted by noise.

- Noise is random in nature and may be described by its Power Spectral Density (PSD).

\[ \text{PSD} = k T_s \]

\[ k = \text{Boltzmann's Constant} = 1.38 \times 10^{-23} \left( \frac{\text{Joules}}{\text{Kelvin}} \right) \]

\[ T_s = \text{System Noise Temperature (Kelvin)} \]
The Radar Equation

- The input noise power for a radar of a given operating bandwidth is then given as
  \[ N_i = PSD \times B = kT_0 \times B \]

- It is always desirable that the minimum detectable signal power be greater than the noise power.

- The fidelity of a radar receiver is normally described by a figure of merit called the noise figure which is defined as
  \[ F = \frac{SNR_i}{SNR_o} = \frac{S_i/N_i}{S_o/N_o} \]

  - \( SNR_i \) = Input Signal to Noise Ratio
  - \( SNR_o \) = Output Signal to Noise Ratio
The Radar Equation

- The input signal power may be rewritten as

\[ S_i = kT_s B F \text{SNR}_o \]

- The minimum detectable signal is then

\[ S_{\text{min}} = kT_s B F (\text{SNR}_o)_{\text{min}} \]

- Setting the detection threshold to the minimum output SNR results in

\[ R_{\text{max}} = \left( \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_s B F (\text{SNR}_o)_{\text{min}}} \right)^{1/4} \]
The Radar Equation

- Rearranging gives the SNR at the output of the receiver.

\[
SNR_o = \frac{P_t G^2 \lambda^2 \sigma}{(4 \pi)^3 k T_s B F L R^4}
\]

- In general, radar losses, \( L \), reduce the overall SNR and thus

\[
SNR_o = \frac{P_t G^2 \lambda^2 \sigma}{(4 \pi)^3 k T_s B F L R^4}
\]
Radar Reference Range

SNR vs Detection Range for Varying Peak Power

SNR vs Detection Range for Varying Target RCS
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