FUNDAMENTALS OF PASSIVE AND ACTIVE SONAR

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ACTIVE SONAR DETECTION MODEL

TRANSMISSION

DELAY AND ATTENUATION

TIME-VARYING MULTIPATH

TARGET

REVERBERATION

AMBIENT NOISE

RECEPTION

TIME-VARYING MULTIPATH

PRE-DETECTION FILTER

DETECTOR

POST-DETECTION PROCESSOR

DECISION: \( H_1/H_0 \) ?
DEFINITIONS

Acoustic pressure, \( p \): The difference between the total pressure, \( p_{\text{total}} \), and the hydrostatic (or undisturbed) pressure \( p_0 \).

\[
\begin{align*}
 p_{\text{total}} &= p_{\text{total}}(x,y,z;t) \\
 p_0 &= p_0(x,y,z) \\
 p &= p(x,y,z;t) = p_{\text{total}} - p_0
\end{align*}
\]
Pascals or Newtons/(meter)^2

Acoustic intensity, \( \vec{I} \): A vector whose component \( \vec{I} \cdot \vec{n} \) in the direction of any unit vector \( \vec{n} \) is the rate at which energy is being transported in the direction \( \vec{n} \) across a small plane element perpendicular to \( \vec{n} \), per unit area of that plane element.

\[
\vec{I} \equiv p \vec{u} ; \quad \text{where } \vec{u} \text{ is the fluid velocity at } (x,y,z;t).
\]

\[
\vec{I} = \vec{I}(x,y,z;t) \text{ Watts/(Meter)}^2
\]
ACOUSTIC INTENSITY LEVELS EXPRESSED IN DECIBELS

For a plane wave, \( I_{\text{ref}} = \frac{(p_{\text{rms}})^2}{\rho c_{\text{ref}}} \)

\( p_{\text{rms}} = \text{Root-mean-square pressure} \)
\( \rho = \text{Density} \)
\( c = \text{Sonic velocity} \)

\( p_{\text{rms, ref water}} = 10^{-6} \text{ Pascals} \)
\( \rho c_{\text{water}} = 1.5 \times 10^6 \text{ kgm/(meter}^2\text{second)} \)

\( p_{\text{rms, ref air}} = 20 \times 10^{-6} \text{ Pascals} \)
\( \rho c_{\text{air}} = 430 \text{ kgm/(meter}^2\text{second)} \)

Intensity level, \( I \), usually refers to a normalized intensity magnitude, \( (\bar{I}_{\text{average}} / I_{\text{ref}}) \), expressed in decibels. For example, \( I = 60 \text{ dB} \) means

\[
60 \text{ dB} = 10 \log_{10} \left( \frac{\bar{I}_{\text{average}}}{I_{\text{ref}}} \right)
\]

\[
6 = \log_{10} \left( \frac{\bar{I}_{\text{average}}}{I_{\text{ref}}} \right)
\]

\[
I = 10^{+6} \times I_{\text{ref}} \text{ [Watts/(meter}^2\text{)]}
\]
ACOUSTIC INTENSITY REFERENCE VALUES: AIR AND WATER

Average acoustic intensity magnitude for a plane wave, \(|\vec{I}| = \frac{(p_{\text{rms}})^2}{\rho c}\).

\(p_{\text{rms}} = \text{Root-mean-square of acoustic pressure}\)

\(\rho = \text{Density}\)

\(c = \text{Sonic velocity}\)

For water: \(I_{\text{ref water}} \equiv |\vec{I}|_{\text{ref water}}\)

\(\rho c = 1.5 \times 10^6 \ \text{kgm}/(\text{meter}^2\text{second})\)

\(I_{\text{ref water}} = [10^{-6} \ \text{Pascals}]^2/ [\rho c]_{\text{water}} = 0.67 \times 10^{-18} \ \text{Watts}/(\text{meter})^2\)

For air: \(I_{\text{ref air}} \equiv |\vec{I}|_{\text{ref air}}\)

\(\rho c = 430 \ \text{kgm}/(\text{meter}^2\text{second})\)

\(I_{\text{ref air}} = [20 \times 10^{-6} \ \text{Pascals}]^2/ [\rho c]_{\text{air}} \approx 10^{-12} \ \text{Watts}/(\text{meter})^2\)
DECIBELS MEASURED IN AIR AND WATER

100 dB \text{ water re } 10^{-6} \text{ Pa} = 10 \log_{10} \left( \frac{X \text{ watts/meter}^2}{I_{\text{ref water}} \text{ watts/meter}^2} \right)

= 10 \log_{10} \left( \frac{X \text{ watts/meter}^2}{6.76 \times 10^{-19} \text{ watts/meter}^2} \right)

100 dB \text{ air re } 20 \times 10^{-6} \text{ Pa} = 10 \log_{10} \left( \frac{Y \text{ watts/meter}^2}{I_{\text{ref air}} \text{ watts/meter}^2} \right)

= 10 \log_{10} \left( \frac{Y \text{ watts/meter}^2}{10^{-12} \text{ watts/meter}^2} \right)

For water: \( X \text{ watts/meter}^2 = 6.76 \times 10^{-9} \text{ watts/meter}^2 \)

For air: \( Y \text{ watts/meter}^2 = 10^{-2} \text{ watts/meter}^2 \)

\[ 10 \log_{10} \left( \frac{10^{-2} \text{ watts/meter}^2}{6.76 \times 10^{-9} \text{ watts/meter}^2} \right) = 61.7 \text{ dB difference in intensity level.} \]
A cross-correlation sequence can be calculated for any pair of causal sequences $x[k]$ and $y[k]$, and there is no implied restriction on their relationship:

$$x_{\text{corr}}[x,y;n] = \sum_{k=-\infty}^{k=+\infty} x[k] y[k-n] = g[n]$$

where $g[n]$ is used to denote a cross-correlation sequence whose constituent sequences are understood but not explicitly stated.

The term ‘replica correlator’ or ‘replica correlation’, is applied when there is some relationship between the input $x[k]$ and the replica $y[k]$, for example:

- $x[k]=y[k]$, i.e, auto-correlation,
- $x[k]$ may be the result of applying a Doppler shift to $y[k]$,
- $x[k]$ is the result of applying a (fixed) shift $n$ to $y[k]$, i.e., $x[k]=y[k-n]$, or
- $x[k] = A y[k-n]$, where $A$ is a constant and the same for all $k$. When this is the case, it is helpful to think of $x[k]$ and $y[k]$ as structurally matched or simply matched.
INPUT (SIGNAL) $x[k]$, $y[k]$

**REPLICA**

- **$k^{(1)}$, FIXED BULK DELAY**
- **$k^{(2)}$ NON-ZERO VALUES OF $x[k]$**
  - $k^{(2)} = 4$
- **$k^{(2)}$ NON-ZERO VALUES OF $y[k]$**
  - $k^{(2)} = 4$

**SHIFTED REPLICA**

- **$n$ INCREASES**
- **$y[k-n]$, $y[k-k^{(1)}]$**

**Aligned with Input**

- **$n = k^{(1)}$**
- **Each $n \geq 0$ produces a different (shifted) $y[k-n]$ sequence.**

**Non-Zero Values**

- **$x[k] = 4$**
- **$y[k] = 4$**

**Additional Notes**

- $n + k^{(2)} = k^{(1)} + k^{(2)}$
The cross-correlation $g[n]$ of $x[k]$ and $y[k]$ is the output of the replica correlator, and

$$g[n] = \sum_{k=0}^{k \geq k^{(1)} + k^{(2)}} x[k]y[k-n]$$

where $n$ is the shift of $y[k]$ with respect to $x[k]$. 
Instead of writing

\[ g[n] = \sum_{k=0}^{k \geq k^{(1)} + k^{(2)}} x[k]y[k-n] \]

we can write

\[ g[n] = \sum_{k=-\infty}^{k = +\infty} x[k]y[k-n] \]

Since \( x[k] = 0 \) if \( k < 0 \) or \( k > k^{(1)} + k^{(2)} \).
The instantaneous power output of the replica correlator $|g[n]|^2$ is calculated for each $n$ and is the detection statistic, i.e., the statistic used to decide if a signal matching, or nearly matching, the replica is embedded in the noise.

A signal is declared to be present if $|g[n]|^2$ exceeds a threshold.
If zero-mean, statistically independent Gaussian noise masks the input signal, cross-correlation of the received data with a replica matching the signal is the optimum receiver structure.
REPLICA CORRELATION FOR A CONTINUOUS CW WAVEFORM (1 of 4)

TRANSMISSION

ECHO

AMBIENT NOISE

RECEIVED DATA

REPLICAS OF THE TRANSMISSION FOR DIFFERENT REPLICA DELAYS

\[ s(t) \]

\[ s(t - \tau_{BD}) \]

\[ s(t - \tau_{BD}) + n(t) \]

\[ s(t - \tau^{(1)}_{RD}) \]

\[ s(t - \tau^{(2)}_{RD}) \]

\[ t = 0 \]
REPLICAS OF THE TRANSMISSION FOR DIFFERENT REPLICA DELAYS

REPLICAS OF THE TRANSMISSION FOR DIFFERENT REPLICA DELAYS

REPLICA CORRELATION FOR A CONTINUOUS CW WAVEFORM (2 of 4)

\[ s(t - \tau_{\text{BD}}) + n(t) \]

\[ s(t - \tau^{(1)}_{\text{RD}}) \]

\[ s(t - \tau^{(2)}_{\text{RD}}) \]

\( t = 0 \)

Shifted replicas, each with the same shape but different delays, \( \tau^{(i)}_{\text{RD}}, i=1,2, \ldots, n, \) are under consideration,

Only one record of the received data is under consideration.
Output of the replica correlator is the product of the echo and the replica for time delays $\tau_d$.

Peak value is at $\tau_d = 0$.

Replica correlator output and its envelope for a CW waveform.

Note triangular shape of upper peaks.

$\tau_d = \tau_{BD} - \tau_{RD}$
Replica correlator output for a CW waveform plus ambient noise.
REPLICA CORRELATOR OUTPUT FOR A CW WAVEFORM MASKED BY NOISE (1 of 3)

Independent normally distributed noise only.

Sine wave 240 samples long with zero-padding on each side.

Normally distributed noise plus sine wave.

Replica of sine wave (offset by -5).

\[ \sigma_{\text{NOISE}} = 4.0 \quad \text{Sine wave amplitude} = 1.0 \]

\[ \frac{\text{Sine wave power}}{\text{Noise power}} = \frac{0.5}{16} = \frac{1}{32} \]

\[ 10 \log(\frac{1}{32}) = -15 \text{ dB} \]
Independent normally distributed noise only.

Sine wave 480 samples long with zero-padding on each side.

Normally distributed noise plus sine wave.

Replica of sine wave (offset -5).

As on previous slide,

Sine wave power
Noise power

$=> -15$ dB
Independent normally distributed noise only.

Sine wave 480 samples long with zero-padding on each side.

Normally distributed noise plus sine wave.

Replica of sine wave (offset -5).

As on previous slide,

\[
\frac{\text{Sine wave power}}{\text{Noise power}} \Rightarrow -15 \text{ dB}
\]
INDEPENDENT NORMALLY DISTRIBUTED NOISE, $\sigma = 1$

SAMPLED DATA POINTS

NOISE VALUES

RANDOM WALK (SUM OF NOISE VALUES UP TO N DATA POINTS)

NUMBER OF STEPS, N

STEPS REMOVED FROM STARTING POSITION

Root-mean-square departure from starting position is $\sqrt{N}$. 
NOISE SAMPLE MULTIPLIED BY A REPLIC

INDEPENDENT NORMALLY DISTRIBUTED NOISE, $\sigma = 1$

POINT-BY-POINT PRODUCTS OF REPLICA AND NOISE
Random walk sample and a general result (2 of 2)

Point-by-point products of replica and noise (from previous slide)

Sampled data points

Product values

Sum of above product values out to N points

Sum of product values

N, number of products summed

N, number of products summed

Root-mean-square departure from starting position (zero) is proportional to $\sqrt{N}$. 
Peak replica correlator output with matched inputs is (nearly) proportional to \( N \), not \( \sqrt{N} \).
If the number of ‘matched’ sampled data points in a received signal and replica is \( N \), and if the noise masking the signal is uncorrelated from sample-to-sample, then:

1) The expectation of the root-mean-square noise output of the replica correlator increases as \( N^{1/2} \),

2) The *peak* output of the replica correlator due to the signal increases (nearly) linearly with \( N \).

3) The ratio of the peak signal output to the root-mean-square noise output is expected to increase as \( \frac{N}{N^{1/2}} \),

4) The peak signal-to-noise *instantaneous power output* of the replica correlator is expected to increase as \( \left( \frac{N}{N^{1/2}} \right)^2 \) = \( N \).
HYPOTHETICAL TRANSMISSION, RECEIVED DATA, AND INSTANTANEOUS POWER OUTPUT OF A REPLICA CORRELATOR

- **REPLICA OF TRANSMISSION FOR TIME SHIFT $\tau_d$**
- **RECEIVED DATA, SIGNAL MATCHING REPLICA PLUS NOISE**
- **OUTPUT OF REPLICA CORRELATOR**
- **OUTPUT AT $\tau_d = 0$ DUE TO SIGNAL ~ $N$**
- **ROOT-MEAN SQUARE VALUE OF NOISE ALONE ~ $N^{1/2}$**
- **INSTANTANEOUS POWER OUTPUT OF REPLICA CORRELATOR AND ITS PEAK POWER ENVELOPE**
  - **MEAN VALUE OF NOISE POWER ALONE ~ $N$** (Too weak to be seen on this scale.)
  - **OUTPUT AT $\tau = 0$ DUE TO SIGNAL ~ $N^2$**
  - **OUTPUT DUE TO NOISE ALONE**

**Arbitrary Units**
THRESHOLD SETTING DETERMINES (WITHIN LIMITS) THE RESULTING PROBABILITIES OF DETECTION AND FALSE ALARM

INSTANTANEOUS POWER OUTPUT OF A REPLICA CORRELATOR FOR A CW WAVEFORM MASKED BY NOISE

TOO HIGH A THRESHOLD LEADS TO MISSED DETECTIONS

TOO LOW A THRESHOLD LEADS TO EXCESSIVE FALSE ALARMS

SAMPLED DATA POINTS FORWARD AND BACKWARD IN TIME FROM EXACT OVERLAP OF CW WAVEFORM AND ITS REPLICA
\( p_d \) is the probability a threshold crossing will occur when a target is actually present.

\( p_{fa} \) is the probability a threshold crossing will occur when a target is not present.

For example, a detector might be designed to provide a 50\% probability of detection while maintaining a probability of false alarm below \( 10^{-6} \).
ACTIVE SONAR DETECTION MODEL
(DISCUSSED SO FAR, ALONG WITH SONAR EQUATION)

TRANSMISSION

DELAY AND ATTENUATION

TIME-VARYING MULTIPATH

FOR CW TRANSMISSIONS

AMBIENT NOISE

TARGET

REVERBERATION

PRE-DETECTION FILTER
(Beamformer)

RECEPTION
(Receive array)

TIME-VARYING MULTIPATH

DELAY AND ATTENUATION

POST-DETECTION PROCESSOR

REPLICA CORRELATOR, DFT DETECTOR

DECISION: $H_1/H_0$?
ACTIVE SONAR DETECTION MODEL
(NEXT STEPS)

TRANSMISSION

DELAY AND ATTENUATION

FOR FREQUENCY MODULATED TRANSMISSIONS

AMBIENT NOISE

RECEPTION (Receive array)

TIME-VARYING MULTIPATH

TARGET

REVERBERATION

DELAY AND ATTENUATION

DECISION: $H_1/H_0$ ?

POST-DETECTION PROCESSOR

REPLICA CORRELATOR, DFT DETECTOR

PRE-DETECTION FILTER (Beamformer)

ACTIVE SONAR DETECTION MODEL
(NEXT STEPS)
Replica correlator output and its envelope for a CW waveform.

Replica matches echo except for time delay.

$\tau_d = 0$

$\tau_d$ is the time delay of the echo with respect to the replica established in the correlator.
EFFECT SHIFTING A CW WAVEFORM’S FREQUENCY BY $\phi$
USING THE ‘NARROWBAND’ APPROXIMATION

$\phi$ is the frequency difference (shift) of the echo with respect to the replica established in the correlator. The greater $|\phi|$, the narrower the envelope of the correlator’s output.
Received CW frequencies \( g_1, g_2, g_3, \ldots, g_6 \) are each the result of the same narrowband Doppler shift with respect to equally spaced transmitted CW frequencies \( f_1, f_2, f_3, \ldots, f_6 \).
TRANSMITTED AND RECEIVED CW FREQUENCIES FOR AN ECHO PRODUCED BY A CLOSING TARGET, A WIDEBAND DOPPLER TRANSFORMATION

Received CW frequencies \( g_1, g_2, g_3, \ldots, g_6 \) are each the result of a different narrowband Doppler shift with respect to equally spaced transmitted CW frequencies \( f_1, f_2, f_3, \ldots, f_6 \).
DOPPLER PARAMETER S AND THE WIDEBAND DOPPLER TRANSFORMATION

- \( s \equiv \frac{(1 - \nu / c)}{(1 + \nu / c)} \equiv 1 - \left(\frac{2\nu}{c}\right) \) if \( \nu / c \ll 1 \)

where

- \( \nu = \) Constant range-rate of a reflector, positive closing, and
- \( c = \) Sonic velocity in the medium.

- Each closing range-rate \( \nu_i \) maps into a ‘stretch’ parameter \( S_i \):
  \[ \nu_i \leftrightarrow S_i \]

- Each transmitted CW pulse of frequency \( f_k \) becomes a received CW pulse of frequency \( g_{ki} \) that depends upon \( S_i \):
  \[ g_{ki} = f_k \left\{ 1 + 2\nu_i / c \right\} = \frac{f_k}{S_i} \quad k = 1, 2, \ldots, N \]

- The difference between the received and transmitted CW frequencies depends on both \( S_i \) and \( f_k \):
  \[ g_{ki} - f_k = f_k \left( 1 - \frac{S_i}{S_i} \right) \]
\( \phi \) is the frequency shift of the echo with respect to the replica established in the correlator. In the ‘wideband’ case the echo undergoes a Doppler transformation and not simply a Doppler shift.
\( \tau_d = 0 \)

\( \tau_d \) is the time delay of the echo with respect to the replica established in the correlator.
$\tau_d$ is the time delay of the echo with respect to the replica established in the correlator.

Time resolution of the envelope is $\sim W^{-1}$ where $W$ is the bandwidth of the waveform.

$\tau_d = 0$

No frequency shift

$\tau_d$
When a frequency-modulated echo experiences a frequency shift with respect to the replica, the peak output of the correlator is diminished and its peak output is no longer at $\tau_d = 0$. 

$\hat{\tau}_d$ is the time delay when the replica correlator’s output envelope is a maximum.

Replica correlator output and its envelope for a frequency-modulated echo experiencing a frequency shift.

With echo frequency shift
THE LINEAR FREQUENCY MODULATED (LFM) WAVEFORM OF UNIT AMPLITUDE

Instantaneous frequency, Hz

Bandwidth, $W$

$-T/2$ to $+T/2$

Time, Seconds

Carrier frequency $f_c$ >> $W$
In the case of the narrowband assumption, $\phi$ is the uniform upward frequency shift of the echo with respect to the replica established in the correlator.
In the wideband case, the echo’s frequency shift is not uniform over its duration, and any frequency shift brings about a dilation or contraction in the duration of the waveform.
Instantaneous frequency (narrowband assumption)

TIME DELAY DIAGRAMS

Echo data moves with respect to replica established in correlator

Increasing clock time

BULK DELAY

$S(t - \tau_{BD})$

$\tau_d > 0$

Clock time

REPLICA DELAY

$S(t - \tau_{RD})$

$\tau_{RD}$

$\tau_{BD}$

$\tilde{t}$

$\tilde{t} = 0$

$\tilde{t}$

$\tilde{t}$

$+\tilde{t}$

$-\tilde{t}$

Time scale referred to replica waveform
TIME DELAY DIAGRAMS

- **Instantaneous frequency** (narrowband assumption)

- **Bulk Delay**

- **Replica Delay**

**Echo data moves with respect to replica established in correlator**

**Increasing clock time**

**Time scale referred to replica waveform**

\[ t = 0 \]

\[ T = \tau_{BD} \]

\[ \tau_d > 0 \]
**TIME DELAY DIAGRAMS**

Echo data moves with respect to replica established in correlator.

Increasing clock time.

---

**Instantaneous frequency (narrowband assumption)**

- $\tilde{t}$
- $\tilde{t}=0$
- $\tilde{t} > 0$

**Time scale referred to replica waveform**

---

**BULK DELAY**

$s(t - \tau_{BD})$

- $t = \tau_{BD}$
- $\tau_d > 0$

---

**REPLICA DELAY**

$s(t - \tau^{(2)}_{RD})$

- $t = 0$
- $t = \tau^{(2)}_{RD}$

---

**Clock time**
Instantaneous frequency (narrowband assumption)

\[ s(t - \tau_{BD}) \]

\[ s(t - \tau_{RD}^{(3)}) \]

Echo data moves with respect to replica established in correlator

Increasing clock time

\[ \tau_d > 0 \]

\[ t = \tau_{BD} \]

\[ t = \tau_{RD}^{(3)} \]

\[ t = 0 \]

Time scale referred to replica waveform

Clock time

T**IME DELAY DIAGRAMS**
\[ \tau_d = \hat{\tau}_d \] when overlap is greatest

**TIME DELAY DIAGRAMS**

- **Instantaneous frequency (narrowband assumption)**
  - \( \tau_d = \hat{\tau}_d \) when overlap is greatest

- **BULK DELAY**
  - \( s(t - \tau_{BD}) \)

- **REPLICA DELAY**
  - \( s(t - \tau_{RD}^{(4)}) \)

- **ECHO**
  - \( t = \tau_{BD} \)

- **REPLICAS**
  - \( t = \tau_{RD}^{(4)} \)

- **Clock time**
  - \( t = 0 \)

Echo data moves with respect to replica established in correlator

Increasing clock time

Time scale referred to replica waveform
**TIME DELAY DIAGRAMS**

- Instantaneous frequency (narrowband assumption)

Echo data moves with respect to replica established in correlator

Increasing clock time

**BULK DELAY**

\[ s(t - \tau_{BD}) \]

**REPLICA DELAY**

\[ s(t - \tau^{(5)}_{RD}) \]

\[ t = 0 \]

\[ t = \tau_{BD} \]

\[ \tau_{d} > 0 \]

\[ t = \tau^{(5)}_{RD} \]
TIME DELAY DIAGRAMS

Echo data moves with respect to replica established in correlator.

Increasing clock time

**Instantaneous frequency (narrowband assumption)**

- $t$ = 0
- $t = \tau_{BD}$

**Bulk Delay**

- $s(t - \tau_{BD})$

**Replica Delay**

- $s(t - \tau_{RD}^{(6)})$

**Time scale referred to replica waveform**

- $\tilde{t} = 0$
- $\tilde{t} = \tau_{RD}^{(6)}$

**Clock time**
TIME DELAY DIAGRAMS

Instantaneous frequency (narrowband assumption)

Echo data moves with respect to replica established in correlator

Increasing clock time

$\tau = \tau_{RD}$

$\tau_d < 0$

$\tau = \tau_{BD}$

$\tau = \tau_{RD}$

$\tilde{t} = 0$

Clock time

BULK DELAY

REPLICA DELAY

REPLICA
USING THE TIME-FREQUENCY DIAGRAM TO ESTIMATE THE MAXIMUM OUTPUT OF A REPLICA CORRELATOR WHEN THE ECHO HAS A FREQUENCY SHIFT $\phi$

The greater the overlap region, the larger the maximum replica correlator output. In this example an LFM waveform has experienced a \textit{narrowband} Doppler \textit{shift}.
In the wideband case, the echo's frequency shift is not uniform over its duration, and any frequency shift brings about a dilation or contraction in the duration of the waveform.
Echo data moves with respect to replica overlapping region producing maximum correlator output.

Here an LFM echo has experienced a wideband Doppler transformation rather than a narrowband Doppler shift. The overlap with the replica is reduced, and the corresponding replica correlator output is reduced.
HYPERBOLIC FREQUENCY MODULATED (HFM) WAVEFORMS, THEIR TIME-PERIOD AND TIME-FREQUENCY DIAGRAMS

Instantaneous period $\tau(t)$, linear

$\Delta \tau$

$-t$ $-T/2$ 0 $+T/2$ $+t$

$\tau_0$ $\tau_1$ $\tau_2$

seconds per cycle

Instantaneous frequency (hyperbolic)

$F_2 = (\tau_2)^{-1}$

$F_1 = (\tau_1)^{-1}$

$W$

$-t$ $-T/2$ 0 $+T/2$ $+t$

HFM waveforms and linear period modulated (LPM) waveforms are the same.
Higher frequency echo data moves with respect to replica overlap region producing maximum correlator output.

Here an HFM echo has experienced a wideband Dopper transformation and the HFM’s time-frequency distribution adjusts itself to remain ‘more’ overlapped than in a similar case for an LFM waveform.
If $U(f)$ is the Fourier transform of waveform $u(t)$, $U(f)$ can be folded over as shown below to obtain Fourier transform $\Psi(f)$.

The inverse Fourier transform of $\Psi(f)$, $\psi(t)$, is a complex waveform in the time domain and provides a convenient way to describe the envelope and phase of the real waveform $u(t)$. $\psi(t)$ is the analytic signal of real waveform $u(t)$, and the Fourier transform of $\psi(t)$ (i.e., $\Psi(f)$) contains no negative frequencies.

\[
\psi(t) \leftrightarrow \text{FT} \rightarrow \Psi(f) \\
\Psi(f) = 2Q(f)U(f) \\
\psi(t) \leftrightarrow \text{FT} \rightarrow \Psi(f)
\]
If \( U(f) \) is the Fourier transform of waveform \( u(t) \), \( U(f) \) can be folded over as show below to obtain Fourier transform \( \Psi(f) \).

The inverse Fourier transform of \( \Psi(f) \), \( \psi(t) \), is a complex waveform in the time domain and provides a convenient way to describe the envelope and phase of the real waveform \( u(t) \). \( \psi(t) \) is the analytic signal of real waveform \( u(t) \).

If \( u(t) \) is a narrowband waveform, the frequency content of \( \psi(t) \), (i.e., \( \Psi(f) \),) is negligible near zero frequency as well as null for all \( f < 0 \).

\[
\Psi(f) = 2 \, Q(f) \, U(f)
\]

\( u(t) \xrightarrow{\text{FT}} U(f) \)

\( \psi(t) \xrightarrow{\text{FT}} \Psi(f) \)
\[ \Psi(f) = 2Q(f)U(f) \]

\[ \psi(t) = u(t) + j \int_{-\infty}^{+\infty} \frac{u(\tau)}{\pi(t-\tau)} \, d\tau \]

\[ u(t) = \text{Re}(\psi(t)) \]

\[ \int_{-\infty}^{+\infty} \frac{u(\tau)}{\pi(t-\tau)} \, d\tau \]

is the Hilbert transform of \( u(t) \) and is often denoted by \( \hat{u}(t) \).
\( \psi(t) \) is the analytic signal of real waveform \( u(t) \), and is always given by

\[
\psi(t) = u(t) + j\hat{u}(t) = u(t) + j\int_{-\infty}^{+\infty} \frac{u(\tau)}{\pi(t-\tau)} d\tau
\]

even if \( u(t) \) is not narrowband! However, it may be difficult to find \( \hat{u}(t) \).

\[
u(t) = -\sin(2\pi f_0 t)
\]
\(-0.5 < t < +0.5, \text{ Seconds}\)
\[
u(t) = 0; \ t > |0.5| \text{ Seconds}
\]
\(f_0 = 3 \text{ Hertz}; \ T = 1 \text{ Second}\)

The envelope of \( u(t) \) is given by:

\[
\sqrt{u^2(t) + \hat{u}^2(t)}
\]
If \( u(t) \) is narrowband, \( \psi(t) \) can be approximated by

\[
\psi(t) \cong \hat{\psi}(t) = [a(t) + j b(t)] \exp[ 2\pi j f_o t ]
\]

where \( a(t) \) and \( b(t) \) are real, \( +f_o \) is a central frequency of \( U(f) \), and \( a(t) \) and \( b(t) \) vary slowly compared to \( 2\pi f_o t \).
ANALYTIC SIGNAL AND COMPLEX ENVELOPE OF REAL WAVEFORM $u(t,f_o)$

$U(f,f_o)$ IS FOURIER TRANSFORM OF $u(t,f_o)$

$\psi(t,f_o)$ IS THE INVERSE FOURIER TRANSFORM OF $\Psi(f,f_o)$

$\nu(t)$ IS THE INVERSE FOURIER TRANSFORM OF $\Psi(f)$

$\nu(t)$ is the complex envelope of real waveform $u(t,f_o)$.

$\psi(t,f_o)$ is the analytic signal of real waveform $u(t,f_o)$. 

$\nu(t)$ is the complex envelope of real waveform $u(t,f_o)$. 

FACTORS OF THE COMPLEX NARROWBAND WAVEFORM $\hat{\psi}(t)$

$\psi(t) = [a(t) + j b(t)] \exp(+2\pi j f_o t)$

$u(t) = \text{Re}(\hat{\psi}(t))$

$a(t)$ and $b(t)$ are real and vary slowly with respect to $2\pi f_o t$.

$[a(t) + j b(t)]$ is the complex envelope of $\hat{\psi}(t)$. 
\[
\hat{\psi}(t) = [a(t) + j b(t)] \exp(+2\pi j f_o t)
\]

\[
\text{Re}[\hat{\psi}(t)] = a(t)\cos(2\pi f_o t) - b(t)\sin(2\pi f_o t)
\]

\[
\text{Im}[\hat{\psi}(t)] = a(t)\sin(2\pi f_o t) + b(t)\cos(2\pi f_o t)
\]

\[
|\hat{\psi}(t)| = \sqrt{a(t)^2 + b(t)^2} = A(t)
\]

\[
\text{Arg}(\hat{\psi}(t)) = \tan^{-1}[\text{Im}[\hat{\psi}(t)] / \text{Re}[\hat{\psi}(t)]] = \Phi(t)
\]

\[
\hat{\psi}(t) = A(t) e^{j[\Phi(t)]}, \quad \Phi(t) = 2\pi f_o t + \theta(t)
\]
DEFINITION OF INSTANTANEOUS FREQUENCY

\[ \hat{\psi}(t) = A(t) e^{j[\Phi(t)]} \]

is a particularly convenient form of the analytic signal because \( \Phi(t) \) is the argument of \( \hat{\psi}(t) \) and the instantaneous phase of \( u(t) \) in radians.

The instantaneous frequency of \( u(t) \) in Hertz is

\[ f_{\text{instantaneous}}(t) = f_i(t) = \frac{1}{2\pi} \frac{d}{dt}[\Phi(t)] \]

Given \( f_i(t) \), the argument of \( \hat{\psi}(t) \) and instantaneous phase of \( u(t) \) is given by

\[ \Phi(t) = 2\pi \int f_i(t) dt + \theta_o \]

And we can write \( \hat{\psi}(t) \) as

\[ \hat{\psi}(t) = A(t) e^{j\left[2\pi \int f_i(t) dt + \theta_o\right]} \]
AMPLITUDE, FREQUENCY, AND PHASE MODULATION

- In the expression $\hat{\psi}(t) = A(t) e^{j \left[ 2\pi \int f_i(t) dt + \theta_o \right]}$
  
  - Variations in $A(t)$ are amplitude modulation or AM
    
    $\left( A(t) = \sqrt{a(t)^2 + b(t)^2} \right)$,
  
  - Variations in $f_i(t)$ are frequency modulation or FM.

- Most active sonar waveforms are either frequency or amplitude modulated; frequency modulation is more common.

- **Phase modulation or PM** is used in underwater communications; for example, $90^\circ$ or $180^\circ$ phase changes are embedded in $\theta(t)$ and the analytic signal is expressed as

  $$\hat{\psi}(t) = A(t) \left[ \exp \left( j (2\pi f_o t + \theta(t)) \right) \right]$$

  i.e., $\theta(t)$ ‘switches’ or ‘flips’ the phase of the carrier frequency.
IMPORTANCE OF THE COMPLEX ENVELOPE

- In the expressions

\[ \hat{\psi}(t) = [a(t) + j b(t)] \exp(+2\pi j f_o t), \text{ and} \]
\[ u(t) = \text{Re}(\hat{\psi}(t)) = a(t)\cos(2\pi f_o t) - b(t)\sin(2\pi f_o t) \]

both the amplitude and frequency modulation of \( u(t) \) and \( \hat{\psi}(t) \) are embedded in the complex envelope \([a(t) + j b(t)] : \)

- Amplitude modulation is \( \sqrt{a(t)^2 + b(t)^2} = A(t) \), and

- Frequency modulation is \( \frac{1}{2\pi} \frac{d[\theta(t)]}{dt} \) where \( \theta(t) = \text{Arg}(a(t)+jb(t)) \)

- All the properties of the waveform (favorable and unfavorable!) independent of \( f_o \) are embedded in the complex envelope.

- This means analyses of most waveform properties (e.g., Doppler tolerance, range resolution) can be carried out by considering only the complex envelope without regard to a ‘carrier’ frequency.
General form of the analytic signal is $\psi(t) = A(t) \ e^{j[\Phi(t)]}$

- $f_{\text{instantaneous}}(t) \equiv f_i(t) \equiv \frac{1}{2\pi} \frac{d}{dt} \left[ \Phi(t) \right]$

Start with $f_i(t)$ for the waveform you want.

Next integrate to find the phase.

Finally, insert $\Phi(t)$ in the general form for $\psi(t)$.

- Variations in $A(t)$ are amplitude modulation or AM
  \[ (A(t) = \sqrt{a(t)^2 + b(t)^2}) \],

- Variations in $f_i(t)$ are frequency modulation or FM.

- $A(t)$ is often explicitly stated, i.e., $\text{rect}(t/T)$.

- $\text{Re}[\psi(t)]$ gives the real waveform, $u(t)$. 

If narrowband.

Need not be narrowband.
OBTAINING THE ANALYTIC SIGNAL AND REAL WAVEFORM

- General form of the analytic signal is \( \psi(t) = A(t) e^{j[\Phi(t)]} \)

Start with \( f_i(t) \) for the waveform you want.

\[ f_{\text{instantaneous}}(t) \equiv f_i(t) \equiv \frac{1}{2\pi} \frac{d}{dt} [\Phi(t)] \]

Then integrate to find the phase.

\[ \Phi(t) = 2\pi \int f_i(t) dt \quad \text{An indefinite integral with a constant of integration.} \]

If narrowband,

\[ \Phi(t) = 2\pi f_o t + \theta(t) + \theta_o \]

where \( d[\theta(t)]/dt \ll 2\pi f_o \)

- Variations in \( A(t) \) are amplitude modulation or AM

\[ (A(t) = \sqrt{a(t)^2 + b(t)^2}) \]

- Variations in \( f_i(t) \) are frequency modulation or FM.

\[ A(t) \text{ is often explicitly stated, i.e., rect}(t/T). \]

\[ \text{Re}[\psi(t)] \text{ gives the real waveform, } u(t). \]
THREE COMMON WAVEFORMS

The following slides give complex representations, i.e., $\psi(t)$ for three common waveforms:

- CW (continuous wave)
- LFM (linear frequency modulation)
- HFM (hyperbolic frequency modulation)
COMPLEX NARROWBAND CW WAVEFORMS OF UNIT AMPLITUDE

\[ u(t) = \text{rect} \left( \frac{t}{T} \right) \begin{align*}
&\xrightarrow{\text{FT}} T \frac{\sin \left( \pi f T \right)}{\pi f T} = U(f) \\
&\xrightarrow{\text{FT}} u(t) \exp \left( 2\pi j f_c t \right) \quad \begin{cases} 
1 & \text{if } |t/T| \leq 1/2 \\
0 & \text{if } |t/T| > 1/2 
\end{cases}
\end{align*} \]

where \[ T f_c \gg 1 \text{ cycle} \]

\[ \psi(t) = \text{rect} \left( \frac{t}{T} \right) \exp \left[ j 2\pi \left( f_c t + \frac{\theta_o}{2\pi} \right) \right] \]

\[ \theta_o \text{ radians is a constant that determines the phase of the CW waveform.} \]

\[ \left( \Phi(t) = 2\pi \int f_c \ dt = 2\pi f_c t + \theta_o \right) \]

\[ \text{Shift to frequency } f_c \text{ to get } U(f - f_c) \]
THE LINEAR FREQUENCY MODULATED (LFM) WAVEFORM OF UNIT AMPLITUDE

Instantaneous phase (radians) \( \Phi(t) = 2\pi f_c t + \theta(t) \), \( t \leq |T/2| \) and \( TW >> 1 \)

and instantaneous frequency (Hertz) \( f_i(t) = \frac{1}{2\pi} \frac{d}{dt}[\Phi(t)] \)

\[ = \frac{1}{2\pi} \frac{d}{dt}[2\pi f_c t + \theta(t)] = f_c + \frac{1}{2\pi} \frac{d}{dt}[\theta(t)] \]

Linear frequency modulation means \( \frac{1}{2\pi} \frac{d}{dt}[\theta(t)] = k t \) so \( \theta(t) = 2\pi \left( \frac{k}{2} t^2 \right) + \theta_o \)

i.e., the instantaneous frequency is a linear function of time, and \( k = W/T \) from the figure.

Therefore the LFM’s instantaneous phase \( = 2\pi \left( f_c t + \frac{k}{2} t^2 + \frac{\theta_o}{2\pi} \right) \) (radians)
THE LINEAR FREQUENCY MODULATED (LFM) WAVEFORM OF UNIT AMPLITUDE

\[ \psi(t) = \text{rect}\left(\frac{t}{T}\right) \exp\left[2\pi j(f_c t + \frac{k}{2} t^2 + \frac{\theta_o}{2\pi})\right] \]

Given: \( \psi(t) = A(t) e^{j[\Phi(t)]} \) and \( A(t) = \text{rect}\left(\frac{t}{T}\right) \)

Step 1: \( f_i(t) = f_c + k t \)

Step 2:
\[
\begin{align*}
\Phi(t) &= 2\pi \int f_i(t) dt \\
\Phi(t) &= 2\pi \left( f_c t + \frac{k}{2} t^2 + \frac{\theta_o}{2\pi} \right)
\end{align*}
\]
HYPERBOLIC FREQUENCY MODULATED (HFM) WAVEFORMS, THEIR TIME-PERIOD AND TIME-FREQUENCY DIAGRAMS

\[
\text{Instantaneous frequency (hyperbolic)} = F_{\text{inst}}(t) = \frac{1}{\tau(t)}
\]

\[
\text{Instantaneous period (linear)} = \tau(t)
\]

HFM waveforms and linear period modulated (LPM) waveforms are the same.
INSTANTANEOUS FREQUENCY FOR AN HFM WAVEFORM OF UNIT AMPLITUDE

\[ \tau_0 = \frac{1}{2} (\tau_1 + \tau_2) = \frac{1}{2} \left( \frac{1}{F_1} + \frac{1}{F_2} \right) \]

\[ \Delta \tau = \tau_1 - \tau_2 = \frac{1}{F_1} - \frac{1}{F_2} \]

\[ \tau(t) = \tau_0 - \frac{t \Delta \tau}{T} \]

\[ W = F_2 - F_1 = \frac{1}{\tau_2} - \frac{1}{\tau_1} \]

\[ F_{\text{inst}}(t) = \frac{1}{\tau(t)} = \frac{T}{T \tau_0 - \Delta \tau t} = \frac{(T/W)F_1F_2}{\frac{1}{2} (T/W) (F_1 + F_2) - t}, \quad -T/2 \leq t \leq +T/2 \]
INSTANTANEOUS FREQUENCY INTEGRATION FOR THE HFM

\[ \psi(t) = A(t) \ e^{j[\Phi(t)]} \text{ and } A(t) = \text{rect}\left[\frac{t}{T}\right] \]

\[ F_{\text{inst}}(t) = \frac{1}{\tau(t)} = \frac{T}{T \tau_o - \Delta \tau t} = \frac{(T/W)F_1F_2}{\frac{1}{2} (T/W) (F_1+F_2) - t}, \quad -T/2 \leq t \leq +T/2 \]

\[ \Phi(t) = 2\pi \int F_{\text{inst}}(t) dt = 2\pi \left\{ \frac{(T/W) F_1 F_2}{\frac{1}{2} (T/W) (F_1+F_2) - t} \right\} + \theta_o \]

\[ \psi(t)_{HFM} = \text{rect}\left[\frac{t}{T}\right] \exp\left[ 2\pi j \left\{ \frac{(T/W) F_1 F_2}{\frac{1}{2} (T/W) (F_1+F_2) - t} \right\} + \frac{\theta_o}{2\pi} \right] \quad -T/2 \leq t \leq +T/2 \]
THE HYPERBOLIC-FREQUENCY-MODULATED (HFM) WAVEFORM OF UNIT AMPLITUDE

\[
\psi(t)_{HFM} = \text{rect}\left[\frac{t}{T}\right] \exp\left[2\pi j \left(\frac{T}{W} F_1 F_2 \ln\left(\frac{1}{\frac{1}{2}(T/W)(F_1 + F_2) - t}\right)\right) + \frac{\theta_0}{2\pi}\right]
\]

\[
\text{rect}(t/T) = \begin{cases} 
1 & \text{if } |t/T| \leq 1/2 \\
0 & \text{if } |t/T| > 1/2 
\end{cases}
\]

\[W = F_2 - F_1, \quad (F_2 > F_1), \quad TW \gg 1, \text{ and } F_1 \gg W\]

\[\theta_0 \text{ radians is a constant that determines the phase of the HFM waveform.}\]
CAUTION

\[ \psi(t)_{HFM} \text{ CANNOT be shifted in time like the rect function, i.e.,} \]

\[ \begin{array}{c}
\text{rect}\left[ \frac{t}{T} \right] \\
\hline
-\frac{T}{2} \quad +\frac{T}{2} \\
-\frac{T}{2} \quad +\frac{T}{2}
\end{array} \]

\[ \text{rect}\left[ \frac{t}{T} - \frac{1}{2} \right] \\
\begin{array}{c}
0 \quad T \\
0 \quad T
\end{array} \]

If \( 0 \leq t \leq T \), the analytic signal for an HFM is

\[ \psi(t)_{HFM} = \text{rect}\left[ \frac{t}{T} - \frac{1}{2} \right] \exp\left[ 2\pi j \left( \frac{T}{W} \right) F_1 F_2 \ln\left( \frac{1}{\frac{1}{2}(T/W)F_2 - t} \right) + \frac{\theta_o}{2\pi} \right] \]

and the above is not a simple time shift of

\[ \psi(t)_{HFM} = \text{rect}\left[ \frac{t}{T} \right] \exp\left[ 2\pi j \left( \frac{T}{W} \right) F_1 F_2 \ln\left( \frac{1}{\frac{1}{2}(T/W)(F_1 + F_2) - t} \right) + \frac{\theta_o}{2\pi} \right] \]

\[-\frac{T}{2} \leq t \leq +\frac{T}{2}\]
ACTIVE SONAR DETECTION MODEL
(NEXT STEPS)

TRANSMISSION

DELAY AND ATTENUATION

TIME-VARYING MULTIPATH

AMBIENT NOISE

RECEPTION

TIME-VARYING MULTIPATH

TARGET

REVERBERATION

DELAY AND ATTENUATION

POST-DETECTION PROCESSOR

PRE-DETECTION FILTER

REPLICA CORR OR DFT

DECISION: $H_1/H_0$ ?
PULSED WAVEFORMS

In general pulses can be:

- 'SHAPED' or UNIFORM,
- OVERLAPPED, or
- NOT OVERLAPPED,

Individual pulses are identified by their carrier frequencies,

However, the frequency content of all pulses is ‘spread’ around the carrier frequency.

Note: $\Delta B$ is the uniform separation distance of the carrier frequencies, not a frequency spread.
The power spectral density of any pulse is ‘spread’ about its carrier frequency: $\Delta F$ is a measure of this spread.
TRANSMITTED AND RECEIVED DURATION TIMES

Impulsive source

Rectangular $\Delta T$ pulse

Opening Doppler

Closing Doppler

$R_1(t)$

$\Delta T$

$(\Delta T_s)$

$s > 1$

$s < 1$

TIME

RANGE

$R_1$
COHERENT AND SEMI-COHOLERT DETECTOR STRUCTURES

For each beamformed channel in, there are $M$ Doppler channels, each corresponding to a discrete, a priori, wideband Doppler hypothesis $S_i$.

**COHERENT PROCESSING:** The output for each Doppler channel $S_i$ is the result of applying the replica matching $g_{1i}, g_{2i}, \ldots, g_{Ni}$ to the entire received signal.

**SEMI-COHOLERT PROCESSING:** The output of each Doppler channel $S_i$ is the result of applying a separate filter operation, i.e., a separate CW replica, to each pulse $g_{1i}, g_{2i}, \ldots, g_{Ni}$ and adding the results.
Detector is a bank of narrowband filters or replicas each centered on different receive CW frequencies $g_{1i}, g_{2i}, \ldots, g_{Ni}$ determined by $s_i$.

Assume target’s Doppler motion produces an echo with stretch factor $s_i$.

Detect is a bank of narrowband filters or replicas each centered on different receive CW frequencies $g_{1i}, g_{2i}, \ldots, g_{Ni}$ determined by $s_i$.
EXPECTED PROCESSING GAIN FOR FULLY COHERENT PROCESSING OVER N PULSES

Processing Gain

NUMBER OF PULSES, N

10 \log_{10} N

Lower frequency

Higher frequency
ACTIVE SONAR DETECTION MODEL
(NEXT STEPS)

1) COMPARISON WITH A 'MATCHED FILTER'
2) AMBIGUITY FUNCTIONS

TRANSMISSION

DELAY AND ATTENUATION

TIME-VARYING MULTIPATH

AMBIENT NOISE

TARGET

REVERBERATION

RECEPTION (Receive array)

TIME-VARYING MULTIPATH

PRE-DETECTION FILTER (Beamformer)

REPLICA CORRELATOR DETECTOR

POST-DETECTION PROCESSOR

DECISION: \( H_1/H_0 \) ?
THE ‘MATCHED FILTER’

- For a linear, time invariant filter:

- The term ‘matched filter’ is applied when the filter’s response function is proportional to the time-reversed input sequence $f[n-k]$ for some value of $n = m$; i.e., when for some shift $m$ of $f[-k]$, $A h[k] = f[m-k]$; then the output at $n = m$ is given by

$$\tilde{g}[m] = \sum_{k=-\infty}^{k=+\infty} h[m-k] f[k]$$

- If $f[n]$ is complex, the ‘matching’ condition is $A h[k] = f^*[m-k]$.

- When the matching condition holds, the filter is essentially cross-correlating the input data $f[n]$ with complex conjugate of the input data.
THE ‘MATCHED FILTER’

- Some authors reserve the term ‘matched filter’ for an analog linear time-invariant filter whose response function is exactly matched to the filter’s time-reversed input function. Common usage relaxes this requirement for ‘exact’ matching – as in the case of replica correlation.

  ➢ For example, when perturbations in time delay and frequency shift affect \( f[k] \), a fixed \( h[k] \) and the same ‘matched filter’ is under consideration even though the perturbed \( f[k] \) is no longer an exact ‘match’ to \( h[k] \).

  ➢ Other detection methods, e.g., the DFT (discrete Fourier transform) should not be confused with the term ‘matched filter’. Neither a replica nor an impulse response is embedded in the DFT.

- From now on we will examine the output of a replica correlator with the understanding that its output is equivalent to a matched filter with a response function obtained by time-reversing the replica established in the correlator.
Using the time-frequency diagram to estimate the maximum output of a replica correlator when the echo has a frequency shift $\phi$.

The greater the overlap region, the larger the maximum replica correlator output. In this example an LFM waveform has experienced a narrowband Doppler shift.
If the echo and replica frequencies achieve partial overlap, the power output of the replica correlator is a local maximum for a fixed $\phi$. The power output is a global maximum when there is complete overlap.
We would like to know the normalized instantaneous power output of a replica correlator (or matched filter) as a function of a waveform’s frequency shift $\phi$ and time delay $\tau_d$ when:

- A replica of the waveform is established in the correlator, and
- The input data (the echo) differs from the replica (the waveform) by only a uniform frequency shift $\phi$ and time delay $\tau_d$. 

NEED FOR A NARROWBAND AMBIGUITY FUNCTION
NARROWBAND AMBIGUITY FUNCTION FOR A TYPICAL LFM WAVEFORM

\[ |\chi(\tau, \phi)|^2 \]
NARROWBAND AMBIGUITY FUNCTION
FOR AN LFM WAVEFORM

\[ T = 1 \text{ sec} \quad W = 10 \text{ Hz} \quad \text{Volume} = 0.99 \]
TRANSMITTED AND RECEIVED DURATION TIMES

Impulsive source

Rectangular $\Delta T$ pulse

Opening Doppler

Closing Doppler
WIDEBAND AMBIGUITY FUNCTION
FOR A TYPICAL HFM WAVEFORM

TIME DELAY IN SAMPLING INTERVALS

AMPLITUDE

Stretch Parameter, S
CW WAVEFORM
DURATION = 0.25 SECONDS
CW FREQUENCY = 3500 Hz

HFM WAVEFORM
DURATION = 0.25 SECONDS
START FREQUENCY = 3450 Hz
END FREQUENCY = 3550 Hz
Time-delay spread and frequency spread degrade the above resolutions when $T$ and $W$ increase beyond limits imposed by these effects!
The correlator output can’t distinguish between a time delay $\tau_o$ and frequency shift $\phi_o$ and zero time delay and zero frequency shift.

$\phi = 0$ corresponds to zero frequency shift.

$\tau = 0$ corresponds to a point target’s bulk delay time.
Source transmission of long CW with stable frequency

Amplitude and Doppler shifted frequency of received echo vary (within limits) at random due to non-stationary medium.
Envelope of transmitted CW waveform

Envelope of echo from closing target is ‘smeared’ on average over a frequency spread of $B$ Hz.

Center frequency of long CW transmission

A similar frequency spread occurs in the absence of a Doppler shift.
Fading of a received signal produced by
medium dispersion on a long CW transmission.

- The amplitude and phase vary (within limits) at random.

- The average duration of a reinforcement
  or fade is \((1 \text{ cycle})/(B \text{ Hz})\) seconds.
CONVOLUTION OF A RECTANGULAR CW’S SINC FUNCTION AND A MEDIUM’S FREQUENCY SPREAD

SINC FUNCTION OF CW TRANSMISSION OF DURATION T.

MEDIUM’S FREQUENCY SPREAD WITH MEAN B

CONVOLUTION OF CW TRANSMISSION AND FREQUENCY SPREAD

f = f_{carrier}

f = f_{carrier}

-\Delta f

\Delta f = 0

+\Delta f

\Delta f = 0

+\Delta f

-\Delta f
TRANSMISSION

TIME

MEDIUM TIME DISPERSION OF TRANSMITTED GAUSSIAN PULSE

EXTENDED TARGET AT CONSTANT RANGE

Slope = c, Sonic velocity in the water

CLOCK
TIME

‘Smearing’ occurs due to extended target; or ‘unresolved’ multipath.

Reception interval is ‘smeared’ on average over a time-delay spread of L seconds.

Bulk time delay (or just ‘time delay’)
Fading of two fixed-frequency received CW echoes in the same acoustic channel at time $t_0$.

Probability received CW echoes at frequencies $f_1$ and $f_2$ will experience local fades or maxima within interval $\Delta T$.

$\Delta T$ is proportional to $L^{-1}$.

$\Delta T$ is the frequency difference $f_1 - f_2$ in Hz.

CW echo $f_2$ experiences a local maximum but echo $f_1$ does not.
EFFECT OF TIME-DELAY & FREQUENCY SPREAD ON AN LFM WAVEFORM, $T_0 = 1$ Sec, $\dot{\omega} = 400$ Hz

Ambiguity Function, $B = 0$ and $L = 0$

Time Delay, $\tau$ Seconds

Frequency Shift, Hertz

$B = 2$ Hz, $L = 5$ ms

Time Delay, $\tau$ Seconds

Frequency Shift, Hertz

dB

0

-10

-20

-30

-40

dB

0

-10

-20

-30

-40
PEAK RESPONSE LOSS FOR LFM WAVEFORM SUBJECTED TO REPLICA CORRELATION

$T_o = 1$ Second, $\omega_0 = 400$ Hz
Recall the results for the ratio of signal output power to interference output power for a replica correlator:

\[
\frac{S}{N} \quad \text{OUTPUT} \quad = \quad \frac{2T_o}{(1 \text{ cycle})} \quad \left\{ \frac{S}{N_o} \right\} \quad \text{INPUT}
\]

when working against ambient noise, and

\[
\frac{S}{R} \quad \text{OUTPUT} \quad = \quad \frac{2W}{(1 \text{ cycle})} \quad \left\{ \frac{S}{R_o} \right\} \quad \text{INPUT}
\]

when working against reverberation.

These results are for coherent processing only, and can be expected to apply within 1 or 2 dB only if:

\[
BT_o < 1 \text{ cycle}
\]

\[
L \cdot W < 1 \text{ cycle}
\]
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