

**Professional Development Short Course On:**  
**Radar Systems Analysis & Design using MATLAB**

**Instructor:**

**Dr. Andy Harrison**

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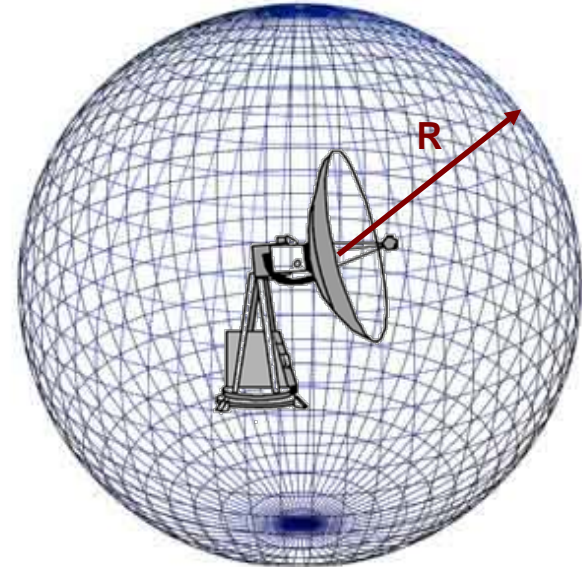
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# The Radar Equation

- Consider a radar with an omnidirectional antenna. The peak power density (power per unit area) at any point in space is

$$P_D = \frac{\text{Transmitted Power}}{\text{Area of a Sphere}} \left( \frac{\text{watts}}{\text{m}^2} \right)$$



- The power density at a given range away from the radar (assuming a lossless propagation medium) is

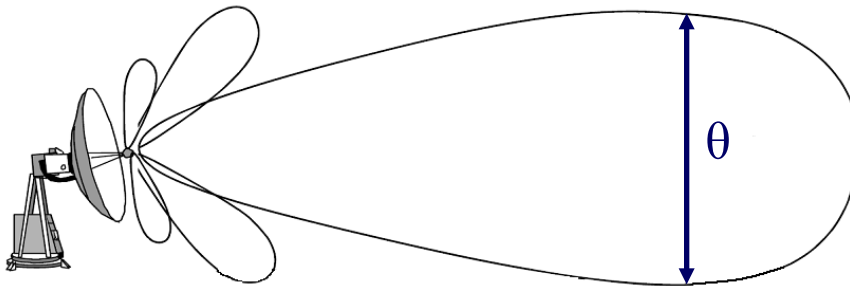
$$P_D = \frac{P_t}{4\pi R^2}$$

$$P_t = \text{Peak Transmitted Power}$$

$$4\pi R^2 = \text{Area of a Sphere of Radius}(R)$$

# The Radar Equation

- ❑ Radar systems utilize directional antennas in order to increase the power density in a certain direction.
- ❑ Directional antennas are usually characterized by the antenna gain and the antenna effective aperture, which are related by



$$G = \frac{4\pi A_e}{\lambda^2}$$

$G$  = Antenna Gain

$A_e$  = Effective Aperture ( $m^2$ )

$\lambda$  = Wavelength ( $m$ )

# The Radar Equation

- The antenna gain is also related to the azimuth and elevation beam widths by

$$G = k \frac{4\pi}{\theta_e \theta_a} \quad k \leq 1$$

$\theta_a$  = Azimuth Beamwidth (radians)

$\theta_e$  = Elevation Beamwidth (radians)

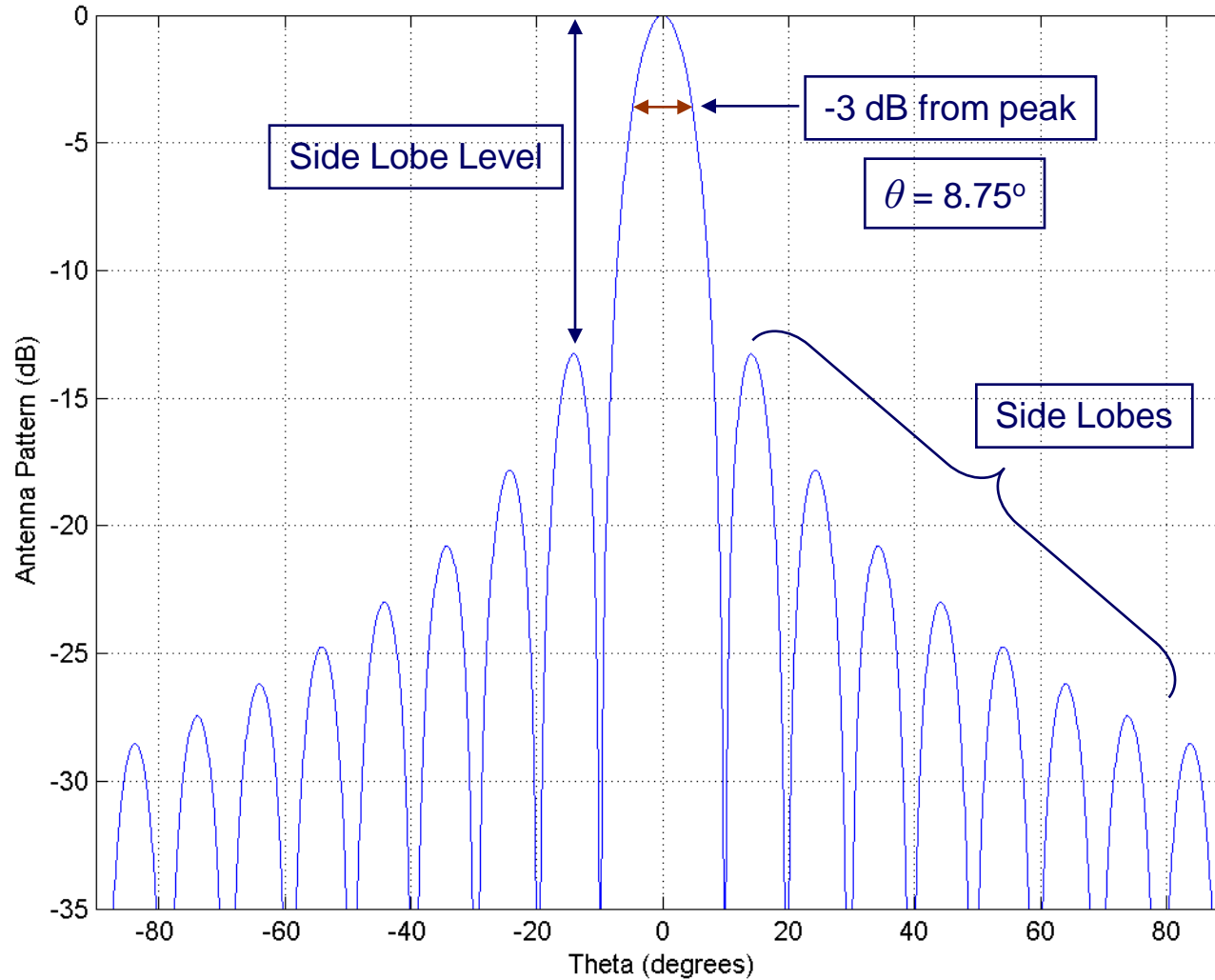
- An accurate approximation for antenna gain is

$$G = \frac{26000}{\theta_e \theta_a} \rightarrow \theta_e \theta_a \text{ (degrees)}$$

- The power density at a given range from the radar using a directive antenna is then given by

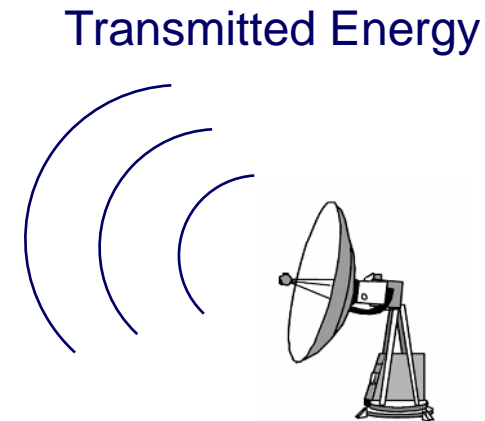
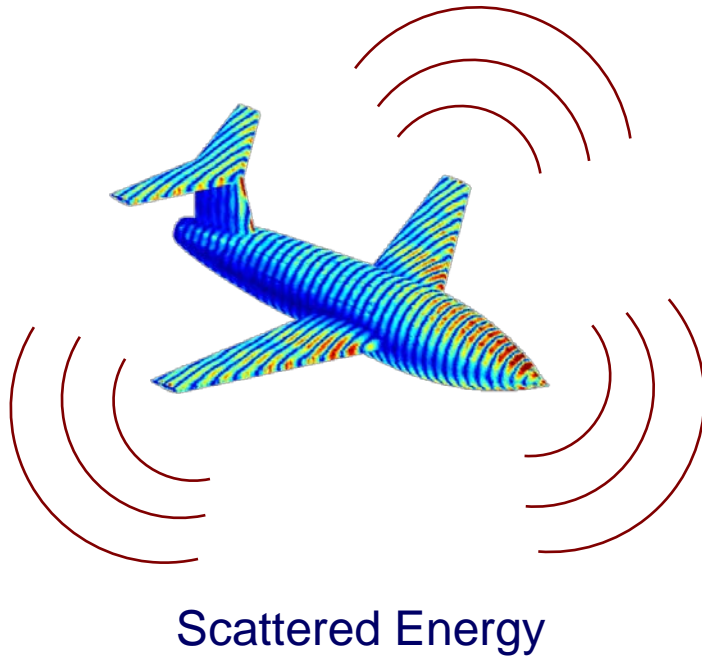
$$P_D = \frac{P_t G}{4\pi R^2}$$

# Antenna Beamwidth



# The Radar Equation

- When the radiated energy from a radar impinges on a target, the induced surface currents on that target re-radiate or scatter electromagnetic energy in all directions.



# The Radar Equation

- ❑ The amount of the scattered energy is proportional to the target size, orientation, physical shape, and material, which are all lumped together in one target-specific parameter called the Radar Cross Section (RCS) and is denoted by  $\sigma$ .
- ❑ The radar cross section is defined as the ratio of the power reflected back to the radar to the power density incident on the target.

$$\sigma = \frac{P_r}{P_D} \quad (m^2)$$

$P_r$  = Reflected Power

$P_D$  = Incident Power Density

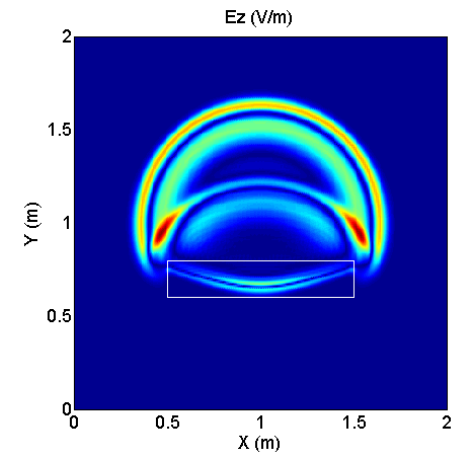
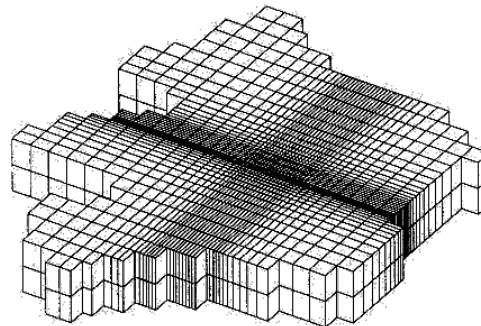
# Signature Modeling Techniques

- ❑ **There are different ways to solve the EM scattering problem. The method used depends on variables such as**
  - **Desired accuracy and appropriateness of the method**
  - **Number of CPUs and amount of RAM available**
  - **Amount of wall time available**
  
- ❑ **The different methods are often grouped together into what are called “low” and “high” frequency computational techniques.**
  
- ❑ **“Low frequency” methods solve the scattering problem in an “exact” sense, incorporating all electromagnetic effects. They are often limited by problem size or computer power.**
  
- ❑ **“High Frequency” methods approximate the scattering problem. They are generally less accurate, though often much faster and less demanding on CPU resources than a low frequency method.**

# Finite Difference Time Domain (FDTD)

- ❑ FDTD solves the time-domain version of Maxwell's Equations. FDTD is considered to be a “full wave” or “exact” method.
- ❑ For scattering problems, the target and some adjoining space must be discretized on a rectangular grid to support the electromagnetic fields.
- ❑ FDTD has high memory and CPU demands, and is not typically used for targets that are electrically large.

$$\begin{aligned}\mu \frac{\partial H_x}{\partial t} &= \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \\ \mu \frac{\partial H_y}{\partial t} &= \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\ \mu \frac{\partial H_z}{\partial t} &= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \\ \epsilon \frac{\partial E_x}{\partial t} &= \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \vec{J}_x \\ \epsilon \frac{\partial E_y}{\partial t} &= \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \vec{J}_y \\ \epsilon \frac{\partial E_z}{\partial t} &= \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \vec{J}_z\end{aligned}$$



3D Maxwell Equations

FDTD Grid

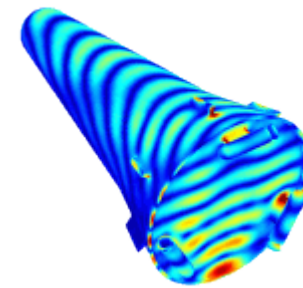
Time-Domain Field

# Method of Moments (MoM)

- ❑ The MoM is used in the scattering problem in the frequency-domain by solving the integral Maxwell's Equations. It is considered to be a “full wave” or “exact” method.
- ❑ The MoM solves for the induced surface current on an object by discretizing the target surface into a number of subdomain “basis functions”. The number of basis functions (N) needed is proportional to the radar wavelength.
- ❑ The MoM creates a matrix equation of order  $N^2$ . This ultimately limits its application to problems of small electrical size.

$$\vec{E}_s = -j\omega\vec{A} - \frac{j}{\epsilon\mu\omega}\nabla\nabla\cdot\vec{A}$$

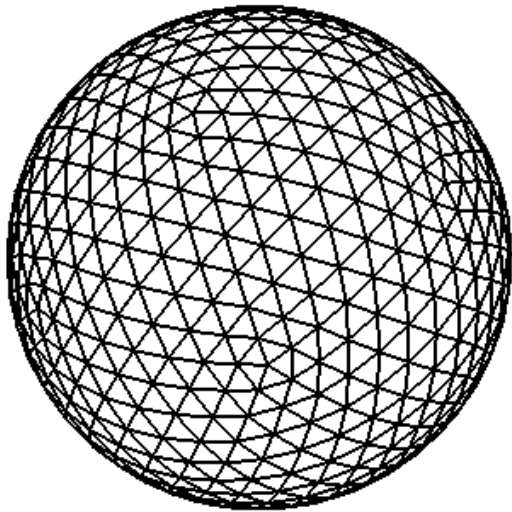
$$4\pi\hat{n}\times\vec{H}^i = 2\pi\vec{J} + \hat{n}\times\iint_S\vec{J}\times\nabla\frac{e^{jkR}}{R}dS$$



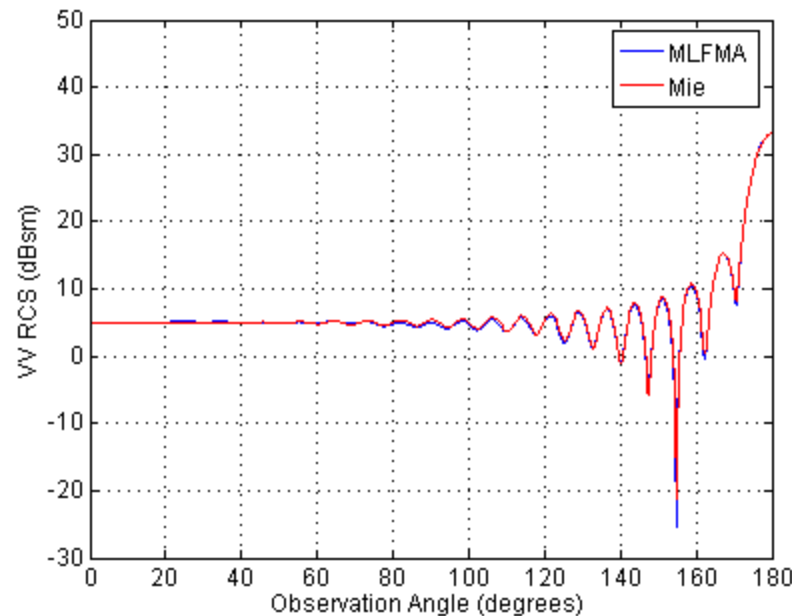
Surface Currents

# Method of Moments (MoM)

- ❑ The Fast Multipole Method (FMM/MLFMA) can be used to reduce the complexity of the MoM matrix system and allow the MoM to be used in problems previously unsolvable.
- ❑ Reliable, industrial-strength FMM software is slowly becoming commercially available. These codes still require supercomputer-class hardware to solve very large MoM problems.



**8 Wavelength Sphere**



**81920 unknowns**

**Regular MoM:**

**50 GB RAM**

**FMM:**

**1.2 GB RAM**

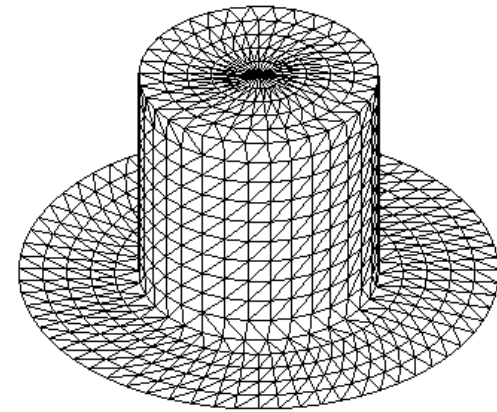
**A factor of 40/1 !**

# Approximate (High-Frequency) Methods

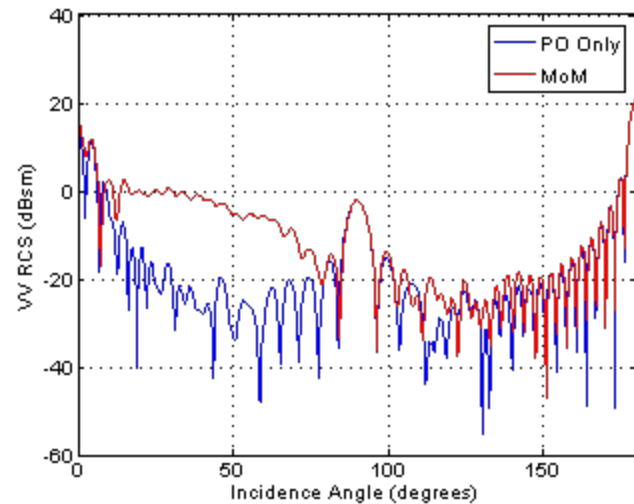
- ❑ “High Frequency” methods approach the scattering problem by assuming the target is very large compared to the radar wavelength.
- ❑ Approximations can then be made to simplify and expedite signature prediction by considering different scattering mechanisms separately.
- ❑ Physical Optics (PO) is a commonly used method that approximates the surface currents by assuming the surface is locally flat. This method does well at speculars but is fair to poor at off-specular angles.
- ❑ The Physical Theory of Diffraction (PTD) supplements PO by accounting for single diffractions from edges.
- ❑ Shooting and Bouncing Rays (SBR) supplements PO and PTD by using ray-optics to model multiple-bounce scattering.
- ❑ Other scattering mechanisms (traveling and creeping waves) can be considered, but are difficult to formulate analytically for general targets.

# Comparison of MoM and Approximate Methods

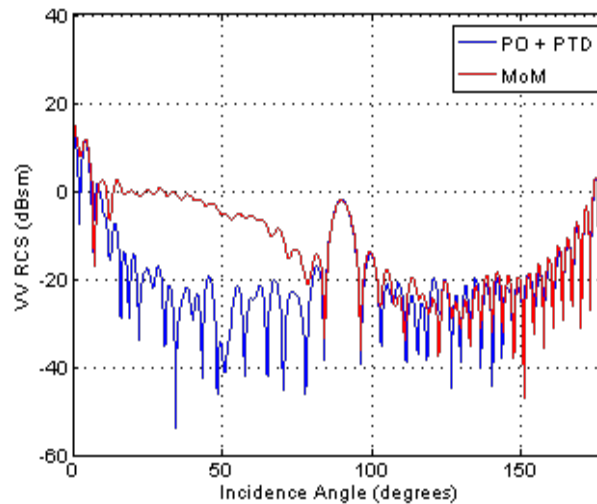
- Target: 16 x 8 inch “Top Hat”
- Monostatic RCS Calculated at 7 GHz
- Target approximately 10 x 5 wavelengths in electrical size.



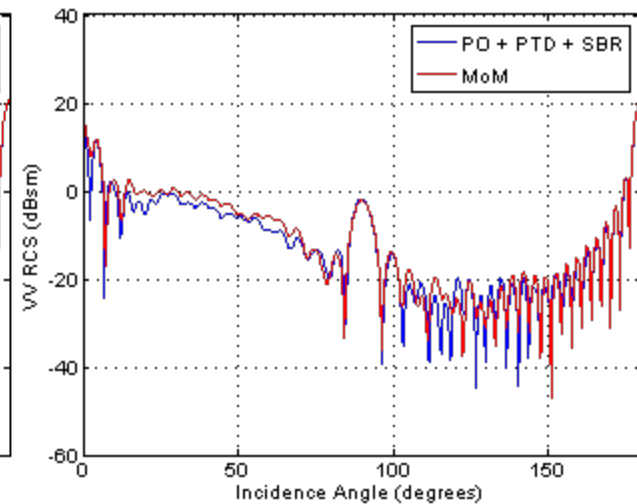
“Top Hat”



PO Only



PO + PTD



PO + PTD + SBR

# Signature Modeling Software

- ❑ **Finite Difference Time Domain (FDTD)**
  - Remcom *xfDTD* (packaging, machines)
  - EMS+ *EZ-FDTD* (packaging, machines)
  
- ❑ **Finite Element Method (FEM)**
  - Ansys, Inc. *ANSYS* (packaging, machines)
  
- ❑ **Method of Moments (MoM)**
  - EM Software and Systems *FEKO* (wire and planar antennas)
  - Tripoint Industries *Serenity* (Scattering by MoM/MLFMA)
  - Tripoint Industries *Galaxy* (Scattering by MoM-BoR)
  
- ❑ **Physical Optics / PTD / SBR**
  - Tripoint Industries *lucernhammer MT* (High-frequency scattering)
  - SAIC *xPatch* (High-frequency scattering)

# The Radar Equation

## □ Typical RCS values of various objects at X-Band.

Object	RCS (m <sup>2</sup> )	RCS (dBsm)
Pickup Truck	200	23
Automobile	100	20
Jumbo Jet	100	20
Commercial	40	16
Cabin Cruiser Boat	10	10
Large Fighter Aircraft	6	7.78
Small Fighter Aircraft	2	3
Adult Male	1	0
Conventional Winged Missile	0.5	-3
Bird	0.01	-20
Insect	1x10 <sup>-5</sup>	-50
Advanced Tactical Fighter	1x10 <sup>-6</sup>	-60

# The Radar Equation

- The power scattered by the target is then

$$P_{\text{target}} = \frac{P_t G \sigma}{4\pi R^2} \quad (\text{watts})$$

- Recalling that the power density at a given range is

$$P_D = \frac{P_t}{4\pi R^2}$$

- Substituting the target's scattered power gives the total power density delivered back to the antenna.

$$P_{\text{antenna}} = \frac{P_t G \sigma}{(4\pi R^2)^2}$$

# The Radar Equation

- ❑ Multiplying by the effective area of the antenna gives the total power received by the radar.

$$P_{radar} = \frac{P_t G \sigma}{(4\pi R^2)^2} A_e$$

- ❑ Substituting for the effective aperture then gives

$$P_{radar} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (\text{watts})$$

# The Radar Equation

□ Denote the minimum detectable signal power by  $S_{\min}$ .

□ It follows that

$$S_{\min} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

□ Rearranging the equation gives the maximum Range to the target.

$$R_{\max} = \left( \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right)^{1/4} \quad (\text{meters})$$

# The Radar Equation

- ❑ In realistic situations, the returned signals received by the radar will be corrupted by noise.
- ❑ Noise is random in nature and may be described by its Power Spectral Density (PSD).

$$PSD = kT_s$$

$$k = \text{Boltzmann's Constant} = 1.38 \times 10^{-23} \left( \frac{\text{Joules}}{\text{Kelvin}} \right)$$

$$T_s = \text{System Noise Temperature} \quad (\text{Kelvin})$$

# The Radar Equation

- ❑ The input noise power for a radar of a given operating bandwidth is then given as

$$N_i = PSD \times B = kT_s \times B$$

- ❑ It is always desirable that the minimum detectable signal power be greater than the noise power.
- ❑ The fidelity of a radar receiver is normally described by a figure of merit called the noise figure which is defined as

$$F = \frac{SNR_i}{SNR_o} = \frac{S_i/N_i}{S_o/N_o}$$

$SNR_i$  = Input Signal to Noise Ratio

$SNR_o$  = Output Signal to Noise Ratio

# The Radar Equation

- The input signal power may be rewritten as

$$S_i = k T_s B F SNR_o$$

- The minimum detectable signal is then

$$S_{\min} = k T_s B F (SNR_o)_{\min}$$

- Setting the detection threshold to the minimum output SNR results in

$$R_{\max} = \left( \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_s B F (SNR_o)_{\min}} \right)^{1/4}$$

# The Radar Equation

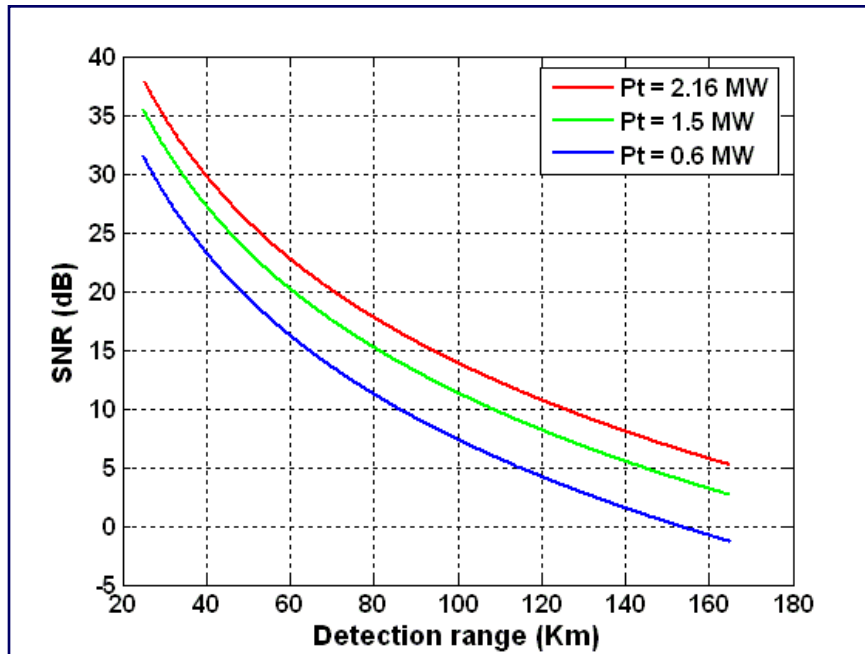
- ❑ Rearranging gives the SNR at the output of the receiver.

$$SNR_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_s B F R^4}$$

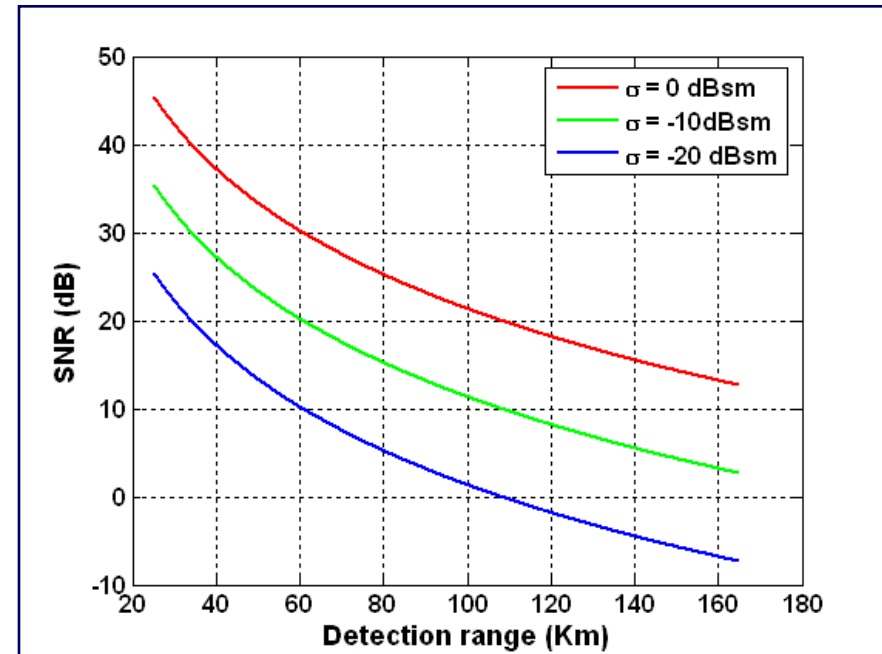
- ❑ In general, radar losses,  $L$ , reduce the overall SNR and thus

$$SNR_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_s B F L R^4}$$

# Radar Reference Range



**SNR vs Detection Range for Varying Peak Power**



**SNR vs Detection Range for Varying Target RCS**

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