Course Sampler From ATI Professional Development Short Course

Attitude Determination & Control

Instructor:

Dr. Mark E. Pittelkau

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FOREWORD

The contents of this book were prepared by the author for the Spacecraft Attitude Determination and Control Course offered through the Applied Technology Institute (ATI). This course material has been continuously revised since its introduction in 1999 to conform to the typical student's needs and to follow technological developments in spacecraft systems, sensors, actuators, and methodologies. Much revision is the direct result of active student participation in the lectures and feedback obtained through the course evaluation form.

Dr. Mark E. Pittelkau

What You Will Learn

This three-and-a-half-day course provides a detailed introduction to spacecraft attitude estimation and control. This course emphasizes many practical aspects of attitude control system design but with a solid theoretical foundation. The student will learn the fundamentals of spacecraft control system engineering. As with any such learning endeavor, the knowledge gained will be retained and strengthened through actual practice.

In this course, spacecraft kinematics and dynamics are developed for use in control design and system simulation. The principles of operation and characteristics of attitude sensors and actuators are discussed. Environmental factors that affect pointing accuracy and attitude dynamics are presented. Pointing accuracy, stability (smear), and jitter definitions, pointing error metrics, and analysis methods are presented. The various types of spacecraft pointing controllers and design, and analysis methods, and back-of-the-envelope design equations are presented. Attitude determination methods are discussed, including TRIAD, QUEST, and Kalman filtering. Sensor alignment and calibration is also covered. The depth and breadth of the topics covered has been adjusted to fit within the alloted time for the course.

There is no specific textbook for this course. However, each section includes a carefully selected bibliography. Many of the references are excellent books.

Students should have an engineering background including calculus and linear algebra. A background in control systems is ideal but not required. A review of control systems theory is included in the course notes, but is not presented due to insufficient time for the course; it would require another half-day. Sufficient background mathematics and control systems theory are presented throughout the course but are kept to the minimum necessary.

About the Instructor

Dr. Mark E. Pittelkau has been an independent consultant since 2005. He was previously with the Applied Physics Laboratory, Orbital Sciences Corporation, CTA Space Systems, and Swales Aerospace. His early career at the Naval Surface Warfare Center involved target tracking, gun pointing control, and gun system calibration, and he has recently worked in target track fusion. His experience in satellite systems covers all phases of design and operation, including conceptual design, implementation, and testing of attitude control systems, attitude and orbit determination, attitude sensor alignment and calibration, optimal slewing, control-structure interaction analysis, stability and jitter analysis, and post-launch support. His current interests are precision attitude determination, attitude sensor calibration, precision attitude control, and optimal slewing. Dr. Pittelkau earned the B.S. and Ph.D. degrees in Electrical Engineering at Tennessee Technological University and the M.S. degree in EE at Virginia Polytechnic Institute and State University.

CONTENTS

KINEMATICS

Overview

- Reference Frames
- Vectors and Vector Operations
- Direction Cosine Matrices
- Rotation Transformations
- Time Derivative of a Vector
- Euler Angles
- Time Derivative of a Direction Cosine Matrix
- Small Angle Transformations
- Quaternions and Quaternion Operations
- Time Derivative of a Quaternion
- Small Angle Quaternions
- Angle-Axis Represenation
- Quaternion $\Leftrightarrow$ DCM Conversion
- Quaternion Transformations of Vectors

Vector Cross Product

- **cross product**: rotation of u into v about axis $\perp$ to u and v

$$\boldsymbol{u} \times \boldsymbol{v} = \begin{bmatrix} u_y v_z - u_z v_y \\ u_z v_x - u_x v_z \\ u_x v_y - u_y v_x \end{bmatrix} = |\boldsymbol{u}||\boldsymbol{v}| \sin\theta\, \mathbf{1}$$

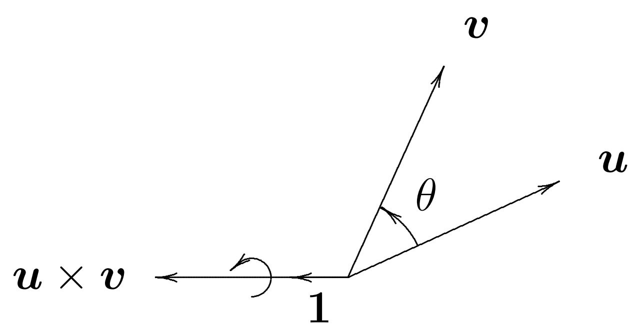
- **cross product matrix**: $[\boldsymbol{u}\times]\boldsymbol{v} = \boldsymbol{u} \times \boldsymbol{v}$

$$[\boldsymbol{u}\times] = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix}$$

- Non-commutativity: $\boldsymbol{u} \times \boldsymbol{v} = -\boldsymbol{v} \times \boldsymbol{u}$
- Products of imaginary units: $\hat{\boldsymbol{\imath}}^2 = \hat{\boldsymbol{\jmath}}^2 = \hat{\boldsymbol{k}}^2 = \hat{\boldsymbol{\imath}}\hat{\boldsymbol{\jmath}}\hat{\boldsymbol{k}} = -1$
- Cross products of basis vectors

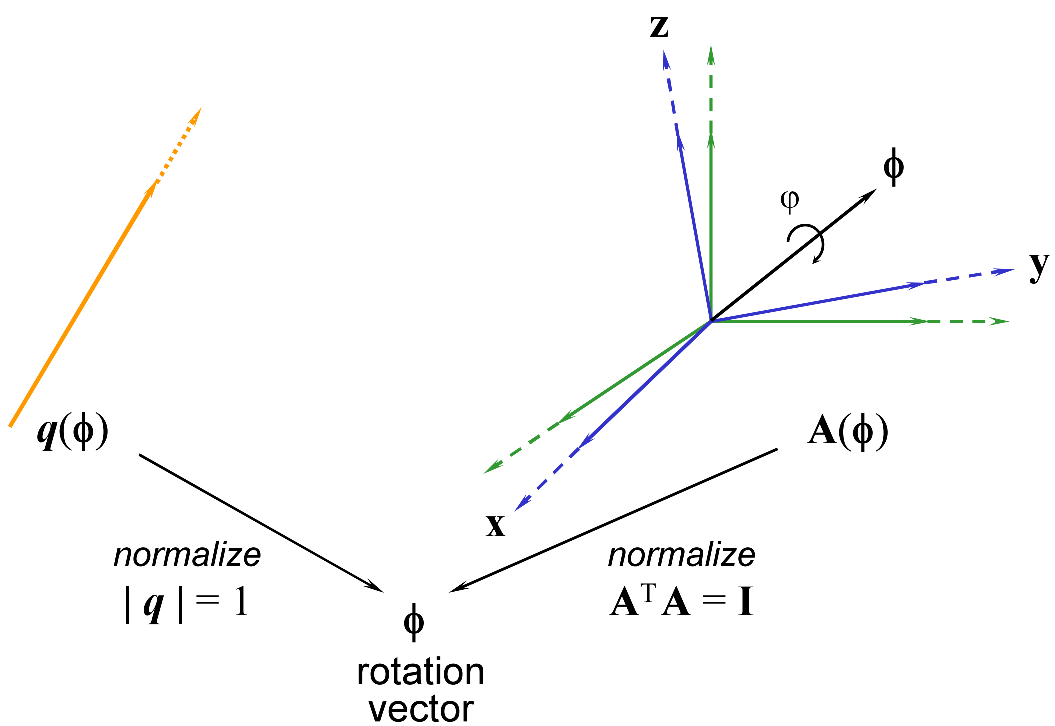
$$\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \times \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = 0\hat{\boldsymbol{\imath}} + 0\hat{\boldsymbol{\jmath}} + 0\hat{\boldsymbol{k}}$$

$$\hat{\boldsymbol{\jmath}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{\imath}} \qquad \hat{\boldsymbol{k}} \times \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \qquad \hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{k}}$$



What is Attitude?

- Attitude (or orientation) is the direction of the axes of a body-fixed frame relative to some other frame. This other frame could be inertially fixed, Earth fixed, etc.
- No matter what rotations resulted in a given attitude, the attitude can be described by a single rotation vector $\boldsymbol{\phi}$ and rotation angle $\varphi = |\boldsymbol{\phi}|$.



$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{r} \\ s \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\boldsymbol{\phi}\,\frac{\sin(\varphi/2)}{\varphi/2} \\ \cos(\varphi/2) \end{bmatrix}$$

$$\mathbf{A}(\boldsymbol{q}) = (s^2 - |\boldsymbol{r}|^2)\mathbf{I} - 2s[\boldsymbol{r}\times] + 2\boldsymbol{r}\boldsymbol{r}^T$$

$$\mathbf{A}(\boldsymbol{\phi}) = (\cos\varphi)\mathbf{I} - \frac{\sin\varphi}{\varphi}[\boldsymbol{\phi}\times] + \frac{1-\cos\varphi}{\varphi^2}\boldsymbol{\phi}\boldsymbol{\phi}^T$$

DYNAMICS

Overview

- Force and Moment
- Inertia Matrix for a Rigid Body
- Generalized Inertia Matrix (Rigid Body)
- Principle Axes of Inertia
- Momentum and Kinetic Energy
- Euler's Equation
- Dynamics of a Spinning Symmetric Body
- Slosh Dynamics
- Wire (Boom) Antennas on Spinning Spacecraft

Moment of Inertia Matrix for a Rigid Body

- Moment of Inertia is also called the Inertia Dyadic or The Inertia Matrix
- Angular velocity of element $\mathrm{d}m$

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r} = -\boldsymbol{r} \times \boldsymbol{\omega}$$

- Angular momentum $\mathrm{d}\boldsymbol{h}$ of mass element $\mathrm{d}m$

$$\mathrm{d}\boldsymbol{h} = \boldsymbol{r} \times (\boldsymbol{v}\,\mathrm{d}m) = -[\boldsymbol{r}\times][\boldsymbol{r}\times]\boldsymbol{\omega}\,\mathrm{d}m$$

- Total angular momentum $\boldsymbol{H}$ of body $\mathcal{B}$

$$\boldsymbol{H} = \int_{\mathcal{B}} \mathrm{d}\boldsymbol{h} = \left[\int_{\mathcal{B}} -[\boldsymbol{r}\times][\boldsymbol{r}\times]\,\mathrm{d}m\right]\boldsymbol{\omega} = \boldsymbol{J}\boldsymbol{\omega}$$

$$\boldsymbol{J} = \int_{\mathcal{B}} -[\boldsymbol{r}\times]^2\,\mathrm{d}m = \int_{\mathcal{B}} \left((\boldsymbol{r}^T\boldsymbol{r})I - \boldsymbol{r}\boldsymbol{r}^T\right)\mathrm{d}m = \int_{\mathcal{B}} \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} \mathrm{d}m$$

- Note negative signs on products of inertia terms (off diagonal elements)
  - *This matrix is sometimes defined without the negative signs. When in doubt, ask!*

Momentum and Kinetic Energy

- Momentum is the phenomenon that keeps a body in motion once that motion is started, assuming there are no perturbing forces or torques
  - Linear momentum: $\boldsymbol{h} = m\boldsymbol{v}$
  - Angular momentum: $\boldsymbol{H} = \boldsymbol{J}\boldsymbol{\omega}$

  These are vector quantities and can be represented in any coordinate system

- Kinetic energy
  - For translational motion: $E_T = \frac{1}{2}m|\boldsymbol{v}|^2$
  - For rotational motion: $E_R = \frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{J}\boldsymbol{\omega}$

- Note that momentum is conserved, energy is not conserved (may be dissipated)
  - Spinning motion about a non-principal axis of inertia eventually becomes motion about the principal axis of inertia ("flat spin")
  - Energy dissipation mechanisms include fuel slosh, antenna and solar array vibration (structural damping), atmospheric friction, damping devices

SENSORS

Overview

- Coarse Sun Sensor (CSS)
- Digital Sun Sensor (DSS)
- Fine Sun Sensor (FSS)
- Static Earth Horizon Sensor (HS)
- Three-Axis Magnetometer (TAM)
- Gyros
  - Types of gyros
  - Error sources
  - Error modeling
  - Allan Variance
- Stellar Inertial Attitude Sensors
  - Star Camera, Star Tracker, Star Scanner
  - Error sources
  - Star catalogs
  - Parallax and Velocity Aberration

Horizon Sensor Errors

- radiance gradient (0.08 to 0.12 deg)
- 15 μm CO_2 altitude uncertainty (30 km (pole in winter) to 40 km (equator))
- Earth oblateness
- detector bias
- calibration table error
- noise

Errors due to radiance gradient may be modeled as first order Markov (correlated) with time constant 500 to 1500 seconds

Optical radius of the Earth at latitude λ given by

$$R = R_\oplus(1 - f\sin^2\lambda + k\sin\lambda) + h$$

where

$R_\oplus$ is the mean equatorial radius of the Earth

f is flattening due to Earth oblateness

h height of the 15 μm IR horizon

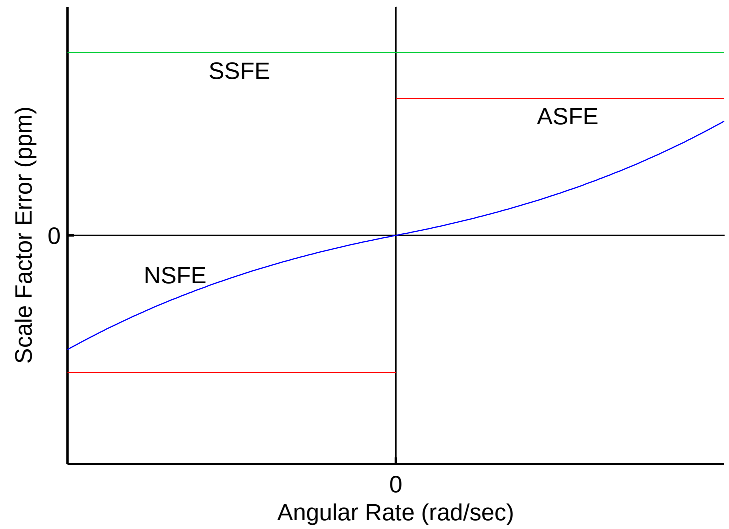
k accounts for seasonal or other latitude-dependent variations

Scale Factor Error

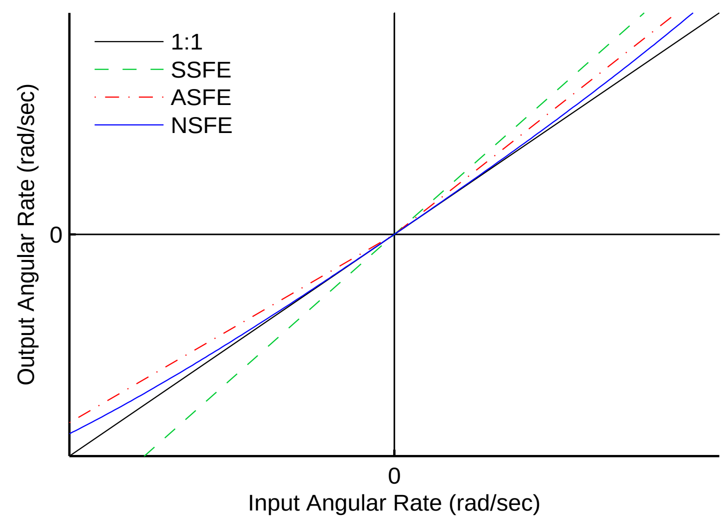
Simple gyro model: $\omega_{\text{out}} = (1 + \text{SFE})K\omega_{\text{in}}$

SFE has three components: $\text{SFE} = \text{SSFE} + \text{ASFE} \cdot \text{sign}(\omega_{\text{in}}) + \text{NSFE}(\omega_{\text{in}})$

ω_{in} — sensed angular rate
ω_{out} — output angular rate
K — is a fixed nominal scale factor
SFE — Scale Factor Error
SSFE — Symmetric Scale Factor Error (can be positive or negative)
ASFE — Asymmetric Scale Factor Error (can be positive or negative)
NSFE — Nonlinear Scale Factor Error, also called scale factor linearity, a nonlinear function of ω_{in}



Types of scale factor error



Deviation from ideal 1:1 transfer function

Actual scale factor nonlinearity may not be such a "nice" function as that shown.
Scale factor errors also change with temperature and ageing.

Low Spatial Frequency Error (LSFE)

Low Spatial Frequency Error (LSFE), sometimes called FOV Rate Spatial Error, varies slowly with location in the FOV. LSFE comprises the following errors:

Optical Distortion Causes star position error to vary with location.

Fixed Focal Length Offset Radial star position error due to focal length error.

Thermal Scale Radial star position error due to focal length change with temperature.

Chromaticity Colors are refracted at slightly different angles as they pass through the lens. They also have different silicon absorption depths in the CCD that results in different spatial responses. Lateral error is compensated based on cataloged star color (spectral class) or B-V index.

Charge transfer inefficiency (CTI, CTE) changes due to radiation degradation, which causes a position dependent centroid error. Even if CTI is compensated, non-uniform CTI produces centroid error.

Calibration Residuals Lens and detector distortion and focal length error may be calibrated but not without residual error.

Fixed Pattern Noise (FPN) is usually caused by timing error or EMI.

Control Systems Review

Overview

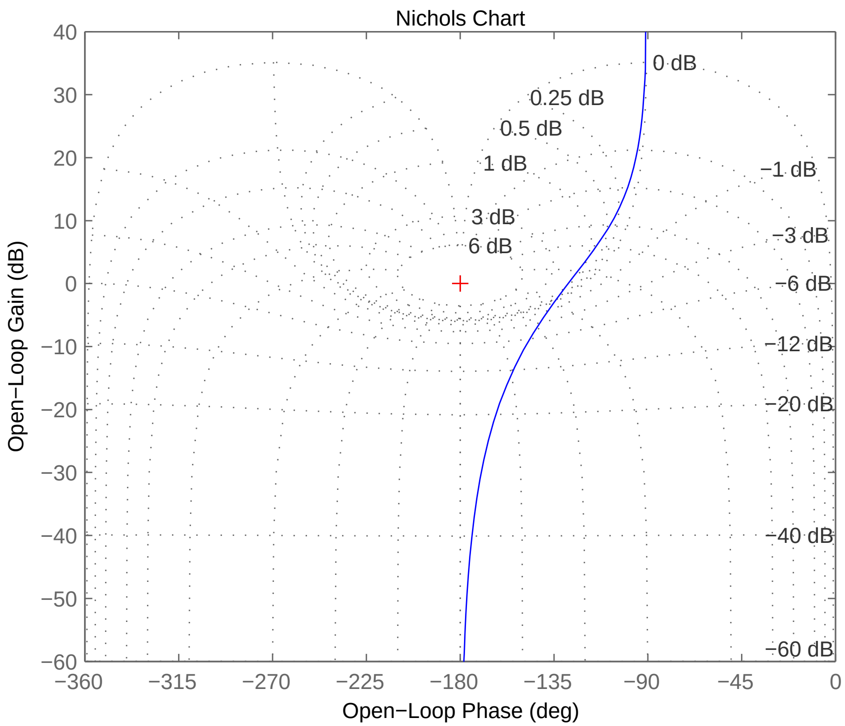
Select Topics from Classical and Modern Control Theory

- System Models via differential equations
- Laplace Transform
- Block Diagrams
- Time Response
- Frequency Response
- Stability (Nyquist, Bode, Nichols plots; M-Circles, Phase and Gain Margins)
- State Space Systems
- State Space Block Diagram
- Response to white noise
- Linear Quadratic Regulator control
- Linear Quadratic Gaussian control
- Stability of LQR and LQG
- Plant Augmentation

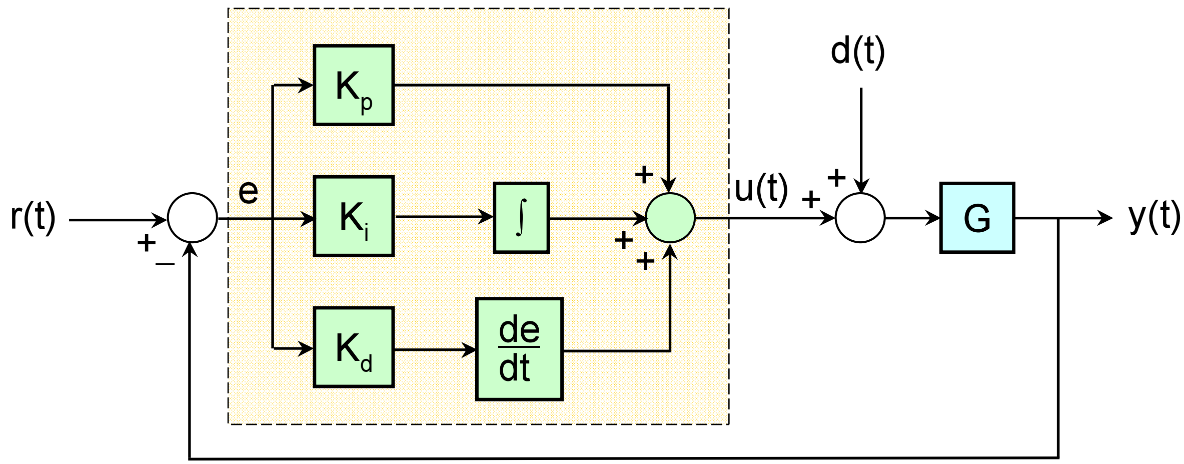
Example Nichols Chart

Previous example with open-Loop Poles on the $j\omega$ axis

$$G(s)K(s) = \frac{1}{s(s+1)}$$



Closed-Loop PID Control System - time domain



K_p	Position gain
K_i	Integral gain
K_d	Derivative gain
G	Plant dynamics (spacecraft dynamics)
$r(t)$	reference or setpoint input (position)
$u(t)$	plant input
$d(t)$	disturbance input
$y(t)$	plant output (position)

Time domain – frequency domain relationships ($s = j\omega$)

$$e(t) \iff E(s) \qquad \frac{\mathrm{d}e(t)}{\mathrm{d}t} \iff sE(s) \qquad \int e(t)\ \mathrm{d}t \iff \frac{1}{s}E(s)$$

SPACECRAFT ATTITUDE CONTROL

Outline

- Implications of orbit/trajectory and mission on ACS design
- Spacecraft Dynamics
- Rate Damping — B-dot and $\boldsymbol{H} \times \boldsymbol{B}$ Laws
- Gravity Gradient Control
- Spin Stabilization
- Momentum Bias Control
- Zero Momentum Control
- Gyroless Attitude Control
- Typical Design Parameters

Spacecraft Dynamics

Euler's equation

$$\dot{\boldsymbol{H}} + \boldsymbol{\omega} \times \boldsymbol{H} = \boldsymbol{\tau}_h + \boldsymbol{\tau}_{gg} + \boldsymbol{\tau}_d$$

$\boldsymbol{H} = J\boldsymbol{\omega} + \boldsymbol{h}_w$	Total momentum
J	Rigid-body inertia matrix
$\boldsymbol{h}_w$	Wheel momentum
$\boldsymbol{\tau}_{gy} = \boldsymbol{\omega} \times \boldsymbol{H}$	Gyroscopic torque
$\boldsymbol{\tau}_h = \boldsymbol{D} \times \boldsymbol{B}$	Momentum control torque (B-dot or $\boldsymbol{H} \times \boldsymbol{B}$)
$\boldsymbol{\tau}_{gg} = 3\omega_o^2(\boldsymbol{r} \times J\boldsymbol{r})$	Gravity gradient torque
$\boldsymbol{\tau}_d$	Disturbance torque
$\boldsymbol{\tau}_c$	Attitude control torque (torque on the spacecraft)

Substitute into Euler's equation

$$J\dot{\boldsymbol{\omega}} + \boldsymbol{\tau}_w + \boldsymbol{\tau}_{gy} = \boldsymbol{\tau}_h + \boldsymbol{\tau}_{gg} + \boldsymbol{\tau}_d$$

Wheel control torque

$$\boxed{\dot{\boldsymbol{h}}_w = \boldsymbol{\tau}_w = \boldsymbol{\tau}_h - \boldsymbol{\tau}_c}$$

Dynamics equation

$$\boxed{J\dot{\boldsymbol{\omega}} = \boldsymbol{\tau}_c - \boldsymbol{\tau}_{gy} + \boldsymbol{\tau}_{gg} + \boldsymbol{\tau}_d}$$

Rate Damping—B-dot Law

- Bdot is a method for reducing momentum without knowledge of body rates.
- A commanded magnetic moment (in A·m^2) is proportional to $\dot{\boldsymbol{B}} = \mathrm{d}\boldsymbol{B}/\,\mathrm{d}t$

$$\boldsymbol{D} = k_d \dot{\boldsymbol{B}} \qquad\qquad k_d > 0$$

- Generated torque is $\quad \boldsymbol{\tau} = \boldsymbol{D} \times \boldsymbol{B} \quad$ (N·m)
- $\dot{\boldsymbol{B}}$ approximated by high-pass filtering or first order differencing the measured $\boldsymbol{B}$ field
  - First-order difference with samples B_k and sample interval T

$$\dot{\boldsymbol{B}}_k = (1/T)(\boldsymbol{B}_k - \boldsymbol{B}_{k-1})$$

$$\boldsymbol{\tau} = (k_d/T)\boldsymbol{B}_k \times \boldsymbol{B}_{k-1}$$

  - For stability and efficiency, must sample $\boldsymbol{B}$ fast enough so that rotation over one sample interval is $\lesssim 30$ degrees
- The momentum $\boldsymbol{H}$ decreases over an orbit as $\boldsymbol{B}$ changes direction, so is less effective at lower inclination orbits and virtually ineffective for equatorial orbits.
- Usually requires at least one orbit to damp (reduce) angular rate

Linearized Spacecraft Dynamics

- Gravity gradient torque is linearized about nadir-pointing attitude in this expression
- δ_x, δ_y, δ_z are small-angle perturbations from a given attitude frame, in this case the LVLH reference frame
- Omit cross-product inertias that multiply δ_x, δ_y, δ_z

x-axis ("Roll")
$$J_x\ddot{\delta}_x - \left[\omega_o h_y - 4\omega_o^2(J_y - J_z)\right]\delta_x - \left[h_y + \omega_o(J_x - J_y + J_z)\right]\dot{\delta}_z + h_z\dot{\delta}_y = \tau_{hx} + \tau_{dx} + \omega_o h_z - \dot{h}_x - 4\omega_o^2 J_{yz}$$

y-axis ("Pitch")
$$J_y\ddot{\delta}_y + 3\omega_o^2(J_x - J_z)\delta_y + \omega_o h_x\delta_x + \omega_o h_z\delta_z - h_z\dot{\delta}_x + h_x\dot{\delta}_z = \tau_{hy} - \tau_{dy} - \dot{h}_y + 3\omega_o^2 J_{xz}$$

z-axis ("Yaw")
$$J_z\ddot{\delta}_z - \left[\omega_o h_y - \omega_o^2(J_y - J_x)\right]\delta_z + \left[h_y + \omega_o(J_x - J_y + J_z)\right]\dot{\delta}_x - h_x\dot{\delta}_y = \tau_{hz} + \tau_{dz} - \omega_o h_x - \dot{h}_z + \omega_o J_{xy}$$

Roll/Yaw Dynamics In State Space Form

J_{yz} and J_{xz} assumed negligible

$$\begin{bmatrix} \dot\delta_x \\ \dot\delta_z \\ \ddot\delta_x \\ \ddot\delta_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & 0 & 0 & a_{34} \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_z \\ \dot\delta_x \\ \dot\delta_z \end{bmatrix} + \frac{1}{J_z} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_z & -b_y \\ b_y & -b_x & 0 \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{J_x} & 0 \\ 0 & \frac{1}{J_z} \end{bmatrix} \begin{bmatrix} \tau_{dx} \\ \tau_{dz} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ [\boldsymbol{\omega}_o \times] & \mathbf{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta} \\ \dot{\boldsymbol{\delta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\omega}_o \end{bmatrix}$$

(pitch rate included here)

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_z \\ \delta_y \\ \delta_z \end{bmatrix} \qquad \boldsymbol{\omega}_o = \begin{bmatrix} 0 \\ -\omega_o \\ 0 \end{bmatrix}$$

$\omega_o =$ orbital rate $\simeq 0.001$ rad/sec for LEO

$$a_{31} = \frac{h_y\omega_o - 4\omega_o^2(J_y - J_z)}{J_x} \qquad a_{34} = \frac{h_y + \omega_o(J_x - J_y + J_z)}{J_x}$$

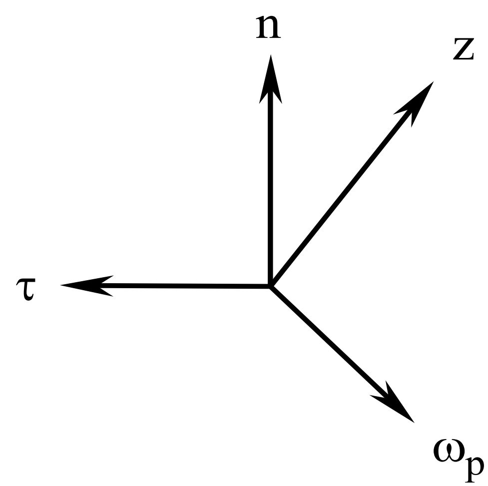
$$a_{42} = \frac{h_y\omega_o - \omega_o^2(J_y - J_x)}{J_z} \qquad a_{43} = -\frac{h_y + \omega_o(J_x - J_y + J_z)}{J_z}$$

Gravity Gradient Effect on Spin Stabilized Spacecraft

Average gravity gradient torque over one orbit

$$\langle \boldsymbol{\tau}_{\text{gg}} \rangle = \frac{3\mu}{\big(a(1-e^2)\big)^3} \big[J_{zz} - (J_{xx} + J_{yy})/2\big] (\boldsymbol{n} \cdot \boldsymbol{z})(\boldsymbol{n} \times \boldsymbol{z})$$

$\boldsymbol{n}$ = unit orbit normal vector
$\boldsymbol{z}$ = unit spin axis vector
μ = gravitational constant
a = semimajor axis
e = eccentricity



GG torque causes the spin axis to precess on a cone about orbit normal with half-cone angle arccos$(\boldsymbol{n} \cdot \boldsymbol{z})$

The rate of precession is proportional to this half-cone angle.

This same effect causes precession of Earth's spin axis with a period of 25,700 years (luni-solar precession).

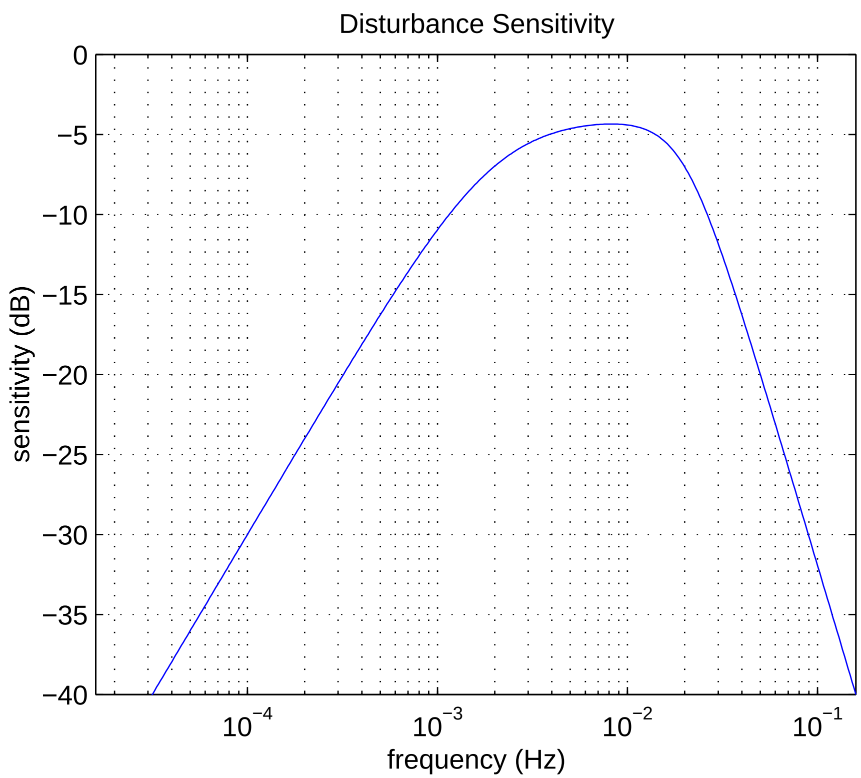
Torque Disturbance Sensitivity

Sensitivity equation: attitude response to torque disturbances (SISO)

$$\frac{Y}{D} = \frac{s/J}{s^3 + K_d s^2 + K_p s + K_i}$$

$\zeta_c = 0.7071$, $\omega_c = (2\pi)0.02$ rad/sec, $a = \omega_c/10$ rad/sec, $J = 100$ kg·m^2

The disturbance sensitivity reaches a peak near ω_c, near the loop bandwidth.



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